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Non-Wellfoundedness

edited by Steph Rennick and Stephan Leuenberger

Contents

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Circular Paths and Infinite Descent
A Guide

Stephan Leuenberger

The contributions to this special issue offer different perspectives on the question whether reality has foundation—whether metaphysical foundationalism is true. The purpose of this introduction is to provide some background. It starts by discussing how metaphysical foundationalism might be characterised—specifically, whether it is committed to the asymmetry, transitivity, or well-foundedness of reality-structuring relations such as parthood, causation, or ground. It then summarises how the articles in the special issue relate anti-foundationalism to the following topics: the history of analytic philosophy, modal epistemology, the relationship between ground and explanation, and between grounds and metagrounds.

1 Introduction to the Introduction

In contemporary metaphysics, it is widely though not uniformly taken for granted that reality has foundations—that there is a fundamental level that gives rise to everything else. A comprehensive world-view is then articulated by telling a story about what the fundamental level is like. A physicalist, for example, may describe the fundamental level as consisting of particles with no further parts, or perhaps of fields. But does such a foundationalist thesis withstand scrutiny? If not, what philosophical lessons could be drawn from the failure of foundationalism? What anti-foundationalist alternatives are there for thinking about the world? These are among the questions that this special issue aims to shed new light on. The papers herein deal, among other things, with the history of anti-foundationalist thinking, the epistemology of possible infinite regresses, the connection between ground and explanation, and potential infinite regresses arising from grounding relationships themselves being grounded.

The more specific aim of this introduction is to provide some background to foundationalism and anti-foundationalism, and to situate the various contributions in the recent literature. We shall be selective in doing so. The above questions have rich and pervasive connections to issues across metaphysics and beyond, and an exhaustive catalogue is beyond the scope of this introduction.

It is customary to understand the foundationalist thesis as concerned with formal features of certain relations. The relations in question are those that impose a hierarchy on their domain, which may contain facts, objects, or events. We shall use "reality-structuring relations" as an umbrella term, with proper parthood, ground, and causation among potential candidates.[1] For each of these relations, and each structural feature, there is an interesting debate about whether the relation has that feature. We shall not engage in such debates but merely survey theoretical options.[2]

After this introduction to the introduction, I shall suggest in section 2 that foundationalism characteristically takes reality-structuring relations to be transitive and irreflexive, and also to satisfy a condition we call "ancestry well-foundedness." In sections 3–4, we discuss anti-foundationalist views that reject respectively one of those three features. We then turn from a sketch of some formal background to introducing the specific themes developed by the contributions to this special issue: Janssen-Lauret's account of Stebbing's antifoundationalist views (section 5); O'Conaill and Pearson's epistemological question to anti-foundationalism (section 6); Billon's and Simsek's explorations of the connection between ground, explanation, and foundationalism (section 7); and Kappes' question whether metaground leads to infinite descent (section 8).

2 Reality-Structuring Relations

We shall not try to give a full account of what it takes for a relation to count as reality-structuring. We do, however, take it to be connected to notions of priority and explanation. If R is a reality-structuring relation that relates x and y, then x is in some sense metaphysically prior to y, or metaphysically

1 Some authors deny that there is a relation of grounding (Correia 2010) or of causation, preferring to express the relevant claims using sentential operators. With suitable higher-order expressive resources, we could recast our discussion in their favoured key.

2 See Dixon (2020) and Bliss and Priest (2018) for taxonomies of pertinent arguments.

explains y.[3] They also need to be fairly natural, as opposed to disjunctive or gerrymandered ones (Lewis 1983; Sider 2011)—a point to which we shall return.

Given a reality-structuring relation R, we can discuss whether foundationalism is true about R. Of course, it might turn out that foundationalism is true about one such relation but not another. Perhaps every object is composed of mereologically atomic parts, while every fact has further grounds.[4] We shall not discuss specific reality-structuring relations here; rather, we are asking what formal features of reality-structuring relations may be taken to capture the foundationalist thesis. Our discussion draws on previous attempts at mapping out this region of conceptual space, including Dixon (2016, 2020, 2023), Rabin and Rabern (2016), and Bliss and Priest (2018). Many of the observations that follow can be found in those authors, though sometimes couched in a different terminology.

The following three formal features are familiar and commonly assumed to hold of proper parthood, partial grounding, and causation:

IRREFLEXIVITY. Not xRx.

ASYMMETRY. If xRy, then not yRx.

TRANSITIVITY. If xRy and yRz, then xRz.[5]

If a relation has all these features, it is a *strict partial order*. As a characterisation of strict partial orders, the list contains some redundancy: asymmetry entails irreflexivity, and irreflexivity and transitivity jointly entail asymmetry. However, we will later explore views that accept some but not all of these theses.

It is also widely held that suitable generalisations of these features hold for mereological composition, full grounding, and joint causation. These relations are not naturally regimented in the form xRy but rather as $xxRy$, with xx a plural variable. The relation R is then collective rather than distributive on

3 It is tempting to add that x will be more fundamental than y. However, it is not clear that this would fit every candidate we shall consider, such as causation. On such issues, see the discussion in Bennett (2017), where a class of "building relations" is characterised.

4 See Raven (2016) for relevant discussion. For the question of how we might read off an overall structure of the world from a multiplicity of reality-structuring relations, see Bennett (2017).

5 The theses are taken to be tacitly universally quantified.

the left (that p and q fully ground r does not imply that p fully grounds r).[6] Apart from a few passing comments, we shall stick to the special case of the non-collective relations, leaving it open what a generalisation might look like.

It is customary to divide anti-foundationalist views about a relation R into coherentist ones that allow loops, or cycles, and infinitist ones that allow infinite descent or abysses.[7] Strict partial orders do not allow any loops and are thus incompatible with coherentism. For *reductio*, suppose that there is a chain $x_1Rx_2, \ldots, x_{n-1}Rx_n, x_nRx_1$. Then by transitivity, x_1Rx_1, in violation of irreflexivity.

However, strict partial orders allow for infinite descent. A foundationalist view that wishes to rule out that possibility will need a further principle. A natural choice is the thesis that R is *well-founded*. The set-theoretic notion of wellfoundedness will need some introduction, though readers familiar with it can skip ahead until the first candidate explication of foundationalism is introduced.

> WELL-FOUNDEDNESS. Every non-empty set S has a member that is R-minimal in S.

An element x of S is said to be *R-minimal* in S just in case there is no y in S such that yRx.

WELL-FOUNDEDNESS entails asymmetry (and hence irreflexivity). For suppose that R is not asymmetric. Then there are x and y such that xRy and yRx. Then $\{x, y\}$ is a non-empty set without an R-minimal member.

For a paradigm of a well-founded strict partial order, consider the relation R_n that holds between x and y just in case they are both natural numbers, and x is smaller than y. Pick any non-empty set S. If the set contains any x that is not a natural number, then there is no y such that yR_nx. Hence x is minimal in S. If S is a set of natural numbers, then it clearly contains a smallest natural number x. Then x will be an R_n-minimal element. So every non-empty S has a member that is R-minimal in S.

For a paradigm of strict partial order that is not well-founded, consider the relation R_i that holds between x and y just in case they are both integers, and x is smaller than y. For every integer x, there is an integer y such that yR_ix.

6 For reasons to prefer the term "left-collective" to the perhaps more familiar "many-one" in this context, see Litland (2018).

7 The terminology of an "abyss" is due to Loss (2016).

Hence the set of all integers is a non-empty set without an element that is R_i-minimal in it.

The relation R_i has an infinite domain—there are infinitely many x such that for some y, either xRy or yRx. Any example of a strict partial order that is not well-founded must share this feature. For suppose that S is a non-empty set without an R-minimal member, with x in S. Then for any n, there is a chain $x_nRx_{n-1}, \dots, x_2Rx_1, x_1Rx$. Since R is transitive and irreflexive, all elements of this chain must be distinct. For any natural number n, then, S has more than n members and thus needs to be infinite. On the other hand, the example of the natural numbers shows that having an infinite domain is only a sufficient and not a necessary condition for a relation being well-founded.

A first candidate explication of foundationalism about a reality-structuring relation R takes it to be the thesis that R is a well-founded partial order. We shall now discuss this candidate explication with regards to the relation of proper parthood and various hypotheses about the mereological structure of the world.

According to what we shall call a *finite particle theory*, there are finitely many particles in the universe, and everything is composed of them. The particles are mereologically atomic: they do not contain any proper parts. However, they are spatially extended. It follows from that theory that proper parthood has a finite domain.[8] So our finite particle theory entails that proper parthood is a well-founded partial order.

Now consider a *gunk theory*, according to which everything has proper parts. Matter is infinitely divisible. Let x be any thing, and consider the set of its proper parts. Since everything has a proper part, that set has no minimal member with respect to proper parthood. So the gunk theory entails that proper parthood is not well-founded.

So far, so good for the provisional explication of foundationalism about R as the thesis that R is a well-founded strict partial order, or equivalently, a well-founded transitive relation. The finite particle theory should clearly count as foundationalist, and the gunk theory as anti-foundationalist, and the explication delivers those verdicts. However, while few have doubted that R's being a well-founded strict partial order is sufficient for foundationalism about R, it has been argued that it is not necessary.

8 If unrestricted mereological composition holds, and there are n atomic particles, the domain of proper parthood will have size $2^n - 1$, which is of course finite if n is.

Consider a *field theory*, which holds that everything is composed of extensionless points. (The familiar fields from physics—graviational, electromagnetic, etc.—are functions defined on such points.) These points do not have proper parts and are thus minimal with respect to proper parthood. But consider the set of objects of non-zero volume. That set is non-empty, and every such object has another one as a proper part. Hence the set has no minimal element with respect to proper parthood, such that proper parthood is not well-founded.

The first explication would thus classify the field theory as anti-foundationalist. This may seem to be the wrong result: for all we have said, our field theory satisfies foundationalist strictures. It seems to capture the thought that everything is determined by the bottom level, consisting of mereological atoms.

For another illustration, consider an *infinite particle theory*. According to that theory, there are infinitely many spatially extended mereological atoms, such that space itself is infinitely extended. Moreover, any plurality of them composes something. Now consider the set of things composed by infinitely many mereological atoms. That set is non-empty, and every member has another one as a proper part. For suppose that x is composed from an infinite plurality of mereological atoms. Then the same plurality minus one is also infinite, and its members will compose a distinct thing y, which is a proper part of x. So the set has no minimal element, and proper parthood fails to be well-founded.

Again, this seems to be the wrong result. Like field theory, infinite particle theory seems intuitively foundationalist. According to both theories, there is infinite descent of proper parthood. But the infinite descent is *bounded below*, in the apt terminology of Rabin and Rabern (2016).

So we may wish to replace well-foundedness with a weaker condition in the explication of foundationalism. (Philosophers who think that 'foundationalism' is merely a term of art, with a definition to be stipulated rather than discovered, may still find it worthwhile to distinguish stronger and weaker conditions on a reality-structuring relation.) A schematic version of a candidate mereological axiom (Simons 1987, 42; Varzi 2016) is a natural choice:

> ATOMICITY. x is R-minimal, or there is an R-minimal y such that yRx.

Here, R-minimality is understood absolutely rather than relative to a given set, as in WELL-FOUNDEDNESS above: x is R-minimal if there is no y such that yRx.[9] If we call an R-minimal element an *atom*, ATOMICITY is equivalent to the claim that for every non-atom x, there is an atom y that stands in R to x.

ATOMICITY is entailed by WELL-FOUNDEDNESS.[10] As desired, the converse does not hold, such that ATOMICITY is strictly weaker. ATOMICITY is of course compatible with the finite particle theory. Unlike WELL-FOUNDEDNESS, it is compatible with the kind of infinite descent that is bounded from below, as exemplified by the field and infinite particle theories. In contrast, ATOMICITY does rule out the gunk theory, on which nothing is minimal with respect to proper parthood. An explication of foundationalism about R by the conditions of IRREFLEXIVITY, TRANSITIVITY, and ATOMICITY thus seems to give the intuitively correct classification of each of our four theories.

ATOMICITY also fails to entail ASYMMETRY or IRREFLEXIVITY, again in contrast to WELL-FOUNDEDNESS. On its own, or supplemented with just transitivity, it is thus compatible with a different kind of scenario we have not yet considered: R-loops that are bounded below, i.e., do not involve things at the bottom of the hierarchy. It is doubtful whether loops of proper parthood that are bounded below are conceptually possible (Kearns 2011). Some theorists of causation, in contrast, have thought that local loops that can be causally accounted for by something outside the loop are less problematic than those that cannot (Lewis 1976, 74).

In the presence of TRANSITIVITY, we can replace ATOMICITY by a weaker condition in this explication, using a new technical term about to be introduced. Doing so will help us generate a more fine-grained taxonomy of anti-foundationalist options later. It will also further illuminate the relationship between WELL-FOUNDEDNESS and ATOMICITY.

For a given object x, let the *R-ancestry* of x be the set of all y such that yRx. So if R is proper parthood, the R-ancestry of x consists of the proper parts of x; if R is partial grounding, the R-ancestry of x consists of the partial

9 ATOMICITY is the restriction to singular argument places on the left of a condition that has been proposed by Dixon (2016) and Rabin and Rabern (2016) to explicate foundationalism about the left-collective relation of full grounding.

10 Suppose that atomicity fails for R. Then there is an x that is not R-minimal, and such that there is no R-minimal y such that yRx. Pick such an x and let S be the set of y such that yRx. Since x is not R-minimal, S is not empty, and since there is no R-minimal y such that yRx, S does not contain any R-minimal element. Hence R is not well-founded.

grounds of x. A set S is an R-ancestry just in case there is an x such that S is the R-ancestry of x. Then consider:

> ANCESTRY WELL-FOUNDEDNESS. Every non-empty R-ancestry set S has a member that is R-minimal in S.

ANCESTRY WELL-FOUNDEDNESS has the same form as WELL-FOUNDEDNESS: it results from inserting "R-ancestry" before "set," which has the effect of restricting the domain of sets quantified over.[11]

While the field theory and the infinite particle theory are incompatible with proper parthood being well-founded, they are perfectly compatible with it being ancestry well-founded. On those theories, the ancestry of a thing, relative to proper parthood, is either empty (if the thing is an atom) or else it includes the atoms it is made up of. The sets that provided counterexamples to WELL-FOUNDEDNESS are not ancestry sets. One way to verify this is by observing that neither includes mereological atoms (points or particles, respectively), and that any ancestry will include mereological atoms if one of these theories is true.

To show that ANCESTRY WELL-FOUNDEDNESS is entailed by ATOMICITY, suppose that S is a non-empty ancestry set. Then there is an x such that S is the ancestry of x. Since S is non-empty, x is not R-minimal. By ATOMICITY, there is an R-minimal y such that yRx. Hence y is in S, and since it is R-minimal, it is *a fortiori* R-minimal in S.

Conversely, we can show that together with TRANSITIVITY, ANCESTRY WELL-FOUNDEDNESS entails ATOMICITY, ensuring that the result of replacing ATOMICITY with ANCESTRY WELL-FOUNDEDNESS in the above explication of foundationalism is equivalent to the original. Suppose that ATOMICITY fails for R. Then there is an x that is not minimal, and such that there is no R-minimal y such that yRx. Consider the ancestry set S of x. Since x is not minimal, S is non-empty. Consider any y in S. Since y is not R-minimal, there is a z such that zRy. Hence zRy and yRx, and by transitivity, zRx. So z is in S too, and it follows that y is not R-minimal in S. Since y was chosen

11 In a generalisation of the concept that can apply to left-collective relations, an R-ancestry can be taken to be a set of pluralities. The key task would be to define a relation R' among pluralities with reference to which minimality is defined. If R is full grounding, R' might hold between X and Y iff X is a weak distributive ground of Y, but not vice versa. (See Fine 2012, 54 for the relevant notion of distributive ground.)

arbitrarily, it follows that S is a non-empty ancestry set without a member that is R-minimal in S.

So on our second attempt, foundationalism about R may be explicated as the conjunction of three claims:

- R is irreflexive.
- R is transitive.
- R is ancestry well-founded.

(To recap: asymmetry is omitted because it is entailed by irreflexivity and transitivity; atomicity because it is entailed by ancestry well-foundedness and transitivity; and well-foundedness because it is arguably not required by foundationalism.)

In the following, we aim to highlight a number of ways in which foundationalism thus understood might fail to be true of a reality-structuring relation. We will discuss a strategy to deflate certain debates concerning formal features by taking them to be essentially verbal.

In so far as the project of explicating (rather than stipulatively defining) a technical term like "foundationalism" makes sense, one may take this proposal to do a reasonably good job. It reflects a typical conception of foundationalism that informs contemporary metaphysical work. As we will see, however, there are reasons to hold that it exaggerates the foundationalist's commitment.

When looking at varieties of metaphysical anti-foundationalism, a natural way to classify them is according to which conjunct they reject. In the following, the focus will be mostly on moderate versions of anti-foundationalism that leave much of the structure intact.

3 Rejecting Transitivity

Foundationalism about a relation R, as we have characterised it, holds that R is transitive, irreflexive, and ancestry well-founded. We might think rejecting the transitivity of reality-structuring relations is a promising way to articulate an anti-foundationalist metaphysics. There are, after all, *prima facie* cases of transitivity failure for certain reality-structuring relations. Several such counterexamples have been proposed for causation (Hall 2000) and for partial grounding (Schaffer 2012). Other putative reality-structuring relations are non-transitive by design. There is increasing recognition of the theoretical need for a non-transitive notion of *immediate* ground (Fine 2012; deRosset 2017;

Werner 2021; Correia 2021). In her list of building relations, Karen Bennett (2017) includes a number of non-transitive ones, notably set formation: x forms the set $\{x\}$—it is its sole member—and $\{x\}$ forms the set $\{\{x\}\}$, but x does not form the set $\{\{x\}\}$, not being a member of it. Accordingly, Bennett rejects the requirement that building relations need to be transitive, in contrast to the conditions of asymmetry and irreflexivity, which she is happy to impose.

Yet debates about foundationalism are not usually seen to hinge on the status of TRANSITIVITY. What we take to be a typical attitude is expressed by Gideon Rosen in his seminal paper on grounding:

> The grounding relation is not obviously transitive, but I shall assume transitivity in a strong form. [...] If the most fundamental relation in the vicinity is not transitive, then [the symbol for grounding] picks out its transitive closure.

Whenever we are talking about a binary relation R, that relation will have a transitive closure R^*—the smallest transitive relation that is implied by R—and we are at liberty to announce that we are talking about R^*. (If R is already transitive, R^* will simply be R itself.)

One way of developing the thought here—perhaps going beyond what Rosen intended—is to take TRANSITIVITY as partly helping us latch onto one referent of "is a ground of" from a pool of potential ones. Other things said by way of explicating the new predicate underdetermine its referent on that conception. Perhaps one candidate is significantly more natural than the others and is thus the referent. But if so, such naturalness comparisons are hardly transparent to us. The satisfaction of TRANSITIVITY is then partly definitive of the relation theorised about. But if so, then it cannot be a substantive claim about a relation reference to which had been independently secured. It might seem to follow that any dispute about the transitivity of grounding is merely verbal.

On that picture, it is possible to proceed as Rosen does and make ground transitive by stipulation. We may wonder, though, whether it is advisable. Given that the predicate introduced is to pick out a reality-structuring relation, would we not wish it to be the most natural one in the neighbourhood? We take that to be an interesting question, but we shall not press it. Perhaps usefulness in metaphysical theorising does not reliably correlate with naturalness. In the realm of genealogy, there is some plausibility to the thought that the parenthood relation is more natural but less theoretically useful than the ancestry relation, which is its transitive closure.

However, there is reason to doubt that questions about the transitivity of reality-structuring relations are typically verbal. Consider David Lewis' first and most influential theory of causation, articulated in Lewis (1973). He first defined a relation of causal dependence between events and then claimed that causation is its transitive closure. Arguably, that move was not prompted by a need to resolve a problem of underdetermination in the expression "causally depends on": that expression had been defined, not just elucidated.[12] Rather, the move is recommended by the pre-theoretical plausibility of the claim that causation is transitive and since it enables Lewis' theory to match the intuitive verdicts in so-called "early preemption" cases. The claim looks as substantial as any in metaphysics, and disagreement about it has not been suspected of being verbal. When the theory was confronted with cases where causation is intuitively not transitive, nobody responded by saying that Lewis' theory is by definition about the transitive closure of the most natural relation in the vicinity. Moreover, whether something is a cause can sometimes make a practical difference that is not merely verbal. Assuming that I am only liable to pay compensation for damage I have caused, whether I am liable may in certain situations turn on whether causation is transitive or not.

The view that ground is transitive by stipulation does not make good sense of how debates about ground are conducted. Rosen's paper itself is a case in point. Before discussing transitivity, he tries to convince us that ground is irreflexive.

> The case for strong irreflexivity is clear enough. Just as no fact can make itself obtain, no fact can play a role along with other facts in making itself obtain.

However, it might happen that the most natural relation in the vicinity is irreflexive, but its transitive closure is not. So if grounding was by stipulation transitive, we would expect to be alerted to that, but we are not.

Lewis' theory of causation is an example of a conflict between irreflexivity and transitivity. By definition, causal dependence relates distinct events only and is thus irreflexive. As Lewis (1986b) notes, though, his theory allows for self-causation. They arise if there are loops of causal dependence, perhaps due to time travel.

12 Maybe there is some underdetermination due to context, but that is presumably an orthogonal issue.

The more general point is that what may look like a stipulation—such as saying that relation R is transitive—can turn out to have substantive implications, specifically for other formal features such as irreflexivity. For that reason, such formal features are best discussed in their interaction rather than one by one.

The question whether a certain reality-structuring relation is transitive is a substantive one and not a verbal one due to one party talking about one relation and the other about its transitive closure. Nonetheless, concerning the specific question whether foundationalism is true about R, it may well be that the transitive closure R^* of R is all we need to consider. The question whether foundationalism is true of a non-transitive relation such as immediate ground, say, is naturally understood as the question whether its transitive closure satisfies certain conditions.[13] So there is a case to modify the explication again. On the third attempt, foundationalism about R is explicated as the conjunction of the following two claims:

- R^* is irreflexive.
- R^* is ancestry well-foundedness.

Clearly, any R satisfying the previous explication also satisfies this one: if R is transitive, then $R = R^*$, and so the irreflexivity of and ancestry well-foundedness of R^* follows from that of R.

The converse does not hold, of course, since R may be non-transitive even if R^* is irreflexive and ancestry well-founded. So the third explication is strictly weaker than the second.

While it may matter greatly whether a given relation is transitive, the truth of foundationalism about R does not hinge on it, if this third explication is right. It appears that if foundationalism is our concern, transitivity is not where the action is, after all.

4 Rejecting Irreflexivity or Ancestry Well-Foundedness

We have noted that the question whether a certain reality-structuring relation is transitive has occasionally been taken to be verbal. The same applies to the question whether such a relation is irreflexive or not. If one philosopher insists that parthood is irreflexive, and another that it is reflexive, it is tempting to conclude that their disagreement is verbal—one using the term

13 Dixon (2023) effectively makes that move.

"part" for what mereologists call "proper part." However, the possibility of verbal disagreement about an issue has no tendency to show that non-verbal disagreement is not also possible.

Again, like in the case of transitivity, we may think that we impose irreflexivity by *fiat*: stipulate that if R is not itself irreflexive, one is referring to its *irreflexive restriction* R^-, which relates x and y iff xRy and $x \neq y$.[14] But we have already seen that we cannot simultaneously ensure transitivity and irreflexivity by *fiat*: by taking the transitive closure of causal dependence, Lewis lost the irreflexivity of causation. Conversely, the irreflexive restriction of a transitive relation need not be transitive.[15]

Does R^* need to be irreflexive for foundationalism about R to be true? Perhaps not. Recall that, as originally conceived, foundationalism rules out both loops and infinite descent. We then weakened the foundationalist ban on infinite descent to allow for descent that is bounded below. This was motivated by considering two toy physical theories, a field theory and an infinite particle theory. We may analogously weaken the ban on loops, allowing them as long as they are bounded below.[16] Since loops of parthood are hard to get one's head around, mereology cannot be expected to supply motivating examples this time. Perhaps there are loops of ground among semantic facts due to self-referential devices in the language, and yet all semantic facts are ultimately grounded in non-semantic facts. If so, loops or cycles arise at the higher levels of reality but not at the bottom level.

In light of the preceding discussion, we arrive at a fourth and even weaker explication of foundationalism about R: as the thesis that R^* is ancestry well-founded.

If R^* is not ancestry well-founded, then either there are circles at the bottom level of reality or there is unbounded infinite descent.[17] The first option

14 Proper parthood is often defined as the irreflexive restriction of parthood. (Alternatively, and equivalently given other assumptions, it is defined as the *asymmetric restriction* of parthood, where the asymmetric restriction R' of R relates x and y iff xRy and not yRx.)

15 The asymmetric restriction of the transitive closure of R is guaranteed to be both transitive and asymmetric (and thus irreflexive). However, it may lack other crucial features, such as non-triviality: if everything forms part of an R-cycle, then $R^{*'}$ will be the empty relation on R's domain.

16 Again, Dixon (2023) deserves credit for articulating this move.

17 We may note that while R^* being ancestry well-founded is necessary for foundationalism about R, R being ancestry well-founded is not. Consider a structure where xRy, yRy, and yRz holds. Then the ancestry of z is the unit set of y, which has no R-minimal element. However, x is the only element at the bottom level, and intuitively, foundationalism is true about R.

has received quite a bit of attention recently (Barnes 2018; McKenzie 2011; Thompson 2016; Calosi and Morganti 2021). Some authors take it to be anti-foundationalist, while others have suggested that it may be compatible with foundationalism (Bennett 2017; Giannotti 2021; Dixon 2023). After all, it does seem to make sense to identify a "bottom level" of reality, consisting of those x such that nothing stands in the *asymmetric restriction* of R to x (i.e., no y is such that yRx but not xRy). In our view, foundationalism ceases to be a distinctive theoretical option if this move is made. However, this is not the place to argue for this.

Recapitulating the four candidate explications of foundationalism about R in reverse order: The final explication requires that R^*—the transitive closure of R—is ancestry well-founded. The penultimate one adds that R^* is irreflexive, ruling out loops bounded below. The antepenultimate explication adds that R is transitive. The most demanding one, which we considered first, adds that R is well-founded. If we wished to allow loops bounded below but not infinite descent bounded below, we could require instead that R^* is ancestry well-founded and that R^{*-}—the irreflexive restriction of the transitive closure of R—is well-founded.

The remainder of the introduction offers summaries of the contributions to this special issue.

5 Anti-foundationalism in the History of Analytic Philosophy

Metaphysical foundationalism seems to have been a commitment of the three men who are often considered the founding fathers of analytic philosophy: Moore, Russell, and Wittgenstein. In "Susan Stebbing on Well-Foundedness," Frederique Janssen-Lauret draws attention to anti-foundationalist elements in the thought of Susan Stebbing, another early analytic philosopher whose work has been neglected until recently. Against some other interpreters, Janssen-Lauret argues that Stebbing did not abandon her method of metaphysical analysis in her mature work. Rather, she gave up the assumption that if there is such a thing as metaphysical analysis, then it must terminate in simples. Whether it does or not is a broadly scientific question, not to be answered a priori. Janssen-Lauret then warns us against understanding Stebbing's metaphysical analysis through the lens of contemporary theoretical posits such as truthmaking or grounding.

6 Anti-foundationalism and Modal Epistemology

Many contemporary philosophers would agree with Stebbing that it is an a posteriori question whether foundationalism is actually true. Much of the debate concerns the metaphysical possibility that the structure of reality might exhibit infinite descent or circularity. We might then take a step back and ask: How can we come to know that a structure is metaphysically possible, other than by inference from its actuality? This question belongs to modal epistemology, or perhaps more accurately the epistemology of possibility. A natural story is broadly recombinatorial: we know that a unicorn is possible because it is possible that a horse and a horn are arranged in a contiguous manner, and the existence of a contiguous arrangement of a horse and a horn grounds the existence of a unicorn.[18] More generally: we establish the metaphysical possibility of non-fundamental p by establishing the metaphysical possibility of a ground of p. Given such a principle, belief in the possibility of infinite regress seems to face the regress of justification familiar from discussion of skepticism. In "Infinite Regresses, Ground Conditions & Metaphysical Satisfaction," Donnchadh O'Conaill and Olley Pearson articulate a principle along these lines, which they call the "Principle of Satisfaction." They use it to argue that we currently lack reasons to think that infinite descent is metaphysically possible.

7 Grounding, Explanation, and Foundationalism

The literature on grounding typically has accepted a rather tight connection between ground and explanation. It is customary to distinguish between "unionist" views that take grounding to be metaphysical explanation and "separatist" views that take grounding to *back* metaphysical explanation.[19] Analogous views have been distinguished in the older debate about causation and causal explanation. However, it may not be plausible that all reality-structuring are explanatory or back explanations. As we have seen, Bennett (2017) does not claim that all building relations are.

How does foundationalism bear on that question? One tempting thought is that it is really the connection to explanation that drives foundationalist intuitions about a certain relation. Explanations cannot go on forever,

18 The second step is controversial (Kripke 1980), but the issues it raises are orthogonal to those of present concern.

19 The labels are introduced in Raven (2015).

and there is no such thing as a circular explanation. This suggests that anti-foundationalists about a given reality-structuring relation need to convince us that widely held views of explanations are wrong, or else revise the connection between that relation and explanation. The relationship between reality-structuring relations and explanation is discussed by two papers in this volume.

In his recent book on infinitism, Ross Cameron (2022) pursues the second option: metaphysical determination relations—roughly what we have called "reality-structuring" relations—need not be explanatory. His argument turns on cases of infinite descent. "Determination Relations and Metaphysical Explanations" criticises Cameron's argument but nonetheless agrees with the conclusion. In the view of Maşuk Şimşek, it is loops, rather than infinite descent, that provide a strong case for divorcing metaphysical determination and explanation.

"A Recipe for Non-wellfounded but Complete Chains of Explanations (And Other Determination Relations)" casts doubt on the widely held view that foundationalism gives us superior explanations to anti-foundationalism. Alexandre Billon works with a conception of ground where grounds do not explain what they ground all by themselves, but in conjunction with certain laws of metaphysics (Schaffer 2017). Given such a conception, it is natural to ask what is explained by the fact that something is grounded according to a law?

Suppose that it is a law that only people without any non-inherited money can inherit, and that if someone inherits, they have one pound less than their testator, or zero, whichever is greater. Can you infer how much money you have from the assumption that you are at the end of the chain of n inheritances, for finite n? No. For any m, your information is compatible with you having m pound, since the starting capital may have been $m + n$. However, from the assumption that your money has been passed down to you through an infinite chain of inheritance, you can infer that you have zero pounds. Given a suitable connection between the ability to infer and explanation, it follows that the existence of an infinite chain is explanatory in a way that the existence of a finite chain is not. So there is a sense in which infinite chains make for superior explanations.

8 Metaground and Infinite Descent

Finally, "Grounding Ground and the (In-)Escapable Ill-Foundedness of the Inclusive 'Explains' " discusses whether there might be an infinitely descending chain of *metagrounds*: grounds of facts that themselves involve ground. The paper makes a number of moves in a short space, so it may not be remiss for my summary of it to be a bit more expansive than that of the other papers. I shall introduce the issue in a simplified form, abstracting away from some subtleties Yannic Kappes considers.

Where f and g are facts, let the fact that g is a partial ground of f be a *link* for f.[20] A sequence of facts is a *link sequence* just in case each successor in the sequence is a link for its predecessor.

It has been widely accepted that links are not fundamental. So they are grounded ($\prec$ expresses partial ground):

LINK-GROUNDED. Any link is grounded.
$g \prec f \rightarrow \exists h(h \prec (g \prec f))$

Suppose that f_2 is a ground of f_1. Then given LINK-GROUNDED, there are facts that form an infinite linking sequence with f_1 as the first element:

$$f_1, f_2 \prec f_1, f_3 \prec (f_2 \prec f_1), f_4 \prec (f_3 \prec (f_2 \prec f_1)), \ldots$$

If we define F_1 as f_1 and F_{i+1} as $f_{i+1} \prec F_i$ for $i > 1$, this sequence can be rewritten as:

$$F_1, F_2, F_3, \ldots$$

LINK-GROUNDED thus entails that if there is an instance of ground at all, there is an infinite linking sequence.

Let a *ground sequence* be a sequence in which each member is grounded by the subsequent one. As a number of authors have pointed out in the literature, none of the generally accepted principles guarantee that the above link sequence is a ground sequence: $F_2 = f_2 \prec f_1$ may not ground $F_1 = f_1$, for example. However, there does seem to be a whiff of infinite descent about that sequence. It is natural to wonder whether its existence guarantees that

20 We could call links "grounding facts." But that term is best avoided due to an ambiguity pointed out by Katherine Hawley (2019): a grounding fact could be a fact is a ground of another fact (as g is, given that g grounds f) or a fact that has the relation of ground as a constituent, i.e., a link such as the fact that g grounds f.

of a closely related infinite grounding sequence. As we are about to see, the following will do the job:

> LINK-GROUND. Any link for f is a ground of f.
> $g \prec f \rightarrow (g \prec f) \prec f$

Consider the element F_{i+1}. It is of the form $f_{i+1} \prec F_i$, where F_i is the predecessor of F_{i+1} in the sequence. Then since $f_{i+1} \prec F_i$ is true, an instance of LINK-GROUND together with modus ponens yields:

$$(f_{i+1} \prec F_i) \prec F_i$$

Or, in rewritten form:

$$F_{i+1} \prec F_i$$

So LINK-GROUND guarantees that every linking sequence is a ground sequence. Together, LINK-GROUNDED and LINK-GROUND entail that grounding is either an empty relation—nothing grounds anything—or else there is an infinitely descending grounding sequence. Have we found a new example of an infinitely descending grounding sequence?

After criticising a pioneering recent discussion of these issues by Frugé (2023), Kappes resourcefully motivates LINK-GROUND—a principle which may not be *prima facie* compelling. This is one of the key contributions of his paper. The other key contribution is his argument that despite being well-motivated, LINK-GROUND ought to be rejected. (We slightly simplify the argument again.) Kappes makes a strong case for the plausibility of the following ($<$ is full ground):

> FULL-LINK. If f is partially grounded in Γ being a partial ground of f, then Γ is not a full ground of f.
> $(\Gamma \prec f) \prec f$, then not $\Gamma < f$.

The idea is that if links are grounds, then full grounds need to include these links. But FULL-LINK is not compatible with LINK-GROUND. Assume that LINK-GROUND holds also when the singular variable g is replaced by a plural variable Γ, and assume further that ground is non-trivial in the sense that there are Γ and f such that $\Gamma < f$. Then, since full grounds are partial grounds, $\Gamma \prec f$. By the generalised form of LINK-GROUND, $(\Gamma \prec f) \prec f$. With FULL-LINK, it follows that not $\Gamma < f$, contradicting our assumption.

FULL-LINK raises interesting questions for further research. It is of the general form of an *exclusion principle* for ground, analogous to exclusion principles that have received a great deal of discussion in connection with mental causation. Such principles are of the form: if Γ is a ground of f, then nothing can be a ground of f unless it is suitably related to Γ. Of course, the suitable relation needs to be spelled out. It is well known that since disjunctive facts may have independent full grounds, formulating tenable exclusion principles for ground is not straightforward.

The completion of this special issue marks the end of the Swiss National Science Foundation research project "Being without Foundations." One of the project aims had been to offer a taxonomy of different varieties of foundationalism and anti-foundationalism. The first part of this introduction has presented relevant building blocks. The summaries of the five articles in the second part of the introduction point towards a reason why an exhaustive taxonomy is not yet to be had at this stage of the debate. As we have seen, the relevant theoretical options depend on a range of background assumptions: about how metaphysical analysis is to be understood (Janssen-Lauret), how possibility facts are established (O'Conaill and Pearson), whether ground is linked to explanation (Şimşek), whether metaphysical laws are among grounds or separate from them (Billon), and whether grounds exclude each other (Kappes). The project question has turned out to be more open-ended than anticipated. If this special issue has pointed towards new ways of tackling it, it will have achieved its aim.*

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Susan Stebbing on Well-Foundedness

Frederique Janssen-Lauret

Susan Stebbing's metaphysical method of directional analysis led her to query the assumption that reality must be well-founded and analysis must terminate in simples. If this is true, she argued, it is a contingent claim about how reality is constituted, not an analytic or logically necessary truth. I present an interpretation of Stebbing's views about well-foundedness, linking her metaphysics to her philosophy of physics. My interpretation evinces that Stebbing did not, as some scholars maintain, abandon directional analysis in the mid-1930s. Instead, she applied it in her philosophy of physics. Stebbing's metaphysical method, I argue, differs in key respects from truth-making, to which it has been compared, and from grounding. Stebbing's metaphysics combines illuminating remarks on well-foundedness with interesting arguments against grounding, which could usefully inform the present-day debate.

Susan Stebbing (1885–1943), until very recently neglected by both historians of analytic philosophy and present-day philosophers, is beginning to be recognized as an important analytic philosopher in her own right (Chapman 2013; Beaney 2016; Janssen-Lauret 2017; Coliva 2021; West 2022), even as a "founding mother" of analytic philosophy (Janssen-Lauret 2024). Here I lay out Stebbing's arguments against the assumption that reality is well-founded, which was commonly held in the early phase of analytic philosophy. Her arguments rest partly on her innovative metaphysical method, using her directional analysis, and partly on her philosophy of physics. I argue that Stebbing's arguments indicate that she did not abandon directional analysis as is commonly held but continued to apply it in her philosophy of physics, and that her arguments still have much to teach us in the twenty-first century.

The canonical "founding fathers" of analytic philosophy, Moore, Russell, and Wittgenstein, all shared at one time or another an assumption that reality is well-founded, that analysis will terminate in simples. Stebbing revealed this assumption to be far shakier than previously supposed by making use of her sharp distinction between "same-level" analysis and "metaphysical" or

"directional" analysis. She argued that while same-level analysis of language in terms of more language may often be analytic or a priori, metaphysical analysis never is. In doing metaphysical analysis, we are concerned with finding out what constituents of reality, in which arrangement, there are in the world if the sentence is true (Stebbing 1932a, 78–80). This can never be an analytic, logically true, or a priori matter because it makes a demand on reality. Reality may or may not be as described. Therefore, we cannot assume that analysis will terminate in simples; this is not a logical, analytic, or a priori truth but one which is beholden to reality being a certain way. It may be false.

Stebbing drew upon her philosophy of physics to argue that we cannot write off metaphysical analyses just because they sound "paradoxical" (Moore 1925, 54), analytically false, or counterintuitive, nor embrace them because they sound intuitive. Modern physics is full of counterintuitive analyses, such as "this solid table is, at the subatomic level, mostly empty space," which we nevertheless have to accept as true. Philosophical analysis has to fit around the findings of physics, not dismiss them. In this way, Stebbing's metaphysics made a key advance on Moore's. She also argued that the new physics tends to suggest arguments against the kind of well-foundedness assumed by the logical atomism of Wittgenstein, whom she interpreted as falling into a form of methodological solipsism at odds with the methods of physics (Stebbing 1933a) and of Russell, who held that analysis terminates in simples with which we are acquainted, a physical impossibility for some subatomic particles (Stebbing 1932a, 72; 1937, 181).

Having explained Stebbing's metaphysics and her proposal regarding the question of well-foundedness, I then compare it to the contemporary range of options on offer and conclude that Stebbing's proposal fares quite well and remains a viable option for us today. Although Stebbing's metaphysical analysis is sometimes compared to truth-making (Egerton 2021), I argue the resemblance is somewhat superficial because truth-making customarily involves some form of determination or necessity. Stebbing, by contrast, did not believe in bespoke metaphysical relations such as determination or metaphysical necessity.

Stebbing's metaphysical analysis might alternatively be read as analysing a fact in terms of further facts, in terms of further facts, *et cetera*, and compared to grounding, with which it shares a chain-like structure. Yet Stebbing spoke out against several argumentative strategies and posits widely embraced by grounding theorists. She argued against the use of "What is it to be a so-and-so?" questions both in metaphysics and in philosophy of science

(Stebbing 1932a, 74–75), against essentialist assumptions and modal discourse involving necessary truth or intensions (Stebbing 1930a, 28, 433), and against reliance on determinative relations in science (Stebbing 1930a, 392–394), in ways that could usefully inform present-day debates. She would also have rejected monism-based solutions to the problem of well-foundedness because she argued that the assumptions that there are multiple perceiving minds and multiple things perceived distinct from the observer are baked into both physics and philosophy from their inception (Stebbing 1929; 1937, 108). Although it is not a logical truth, ontological pluralism cannot in practice be intelligibly denied, according to Stebbing.

In Stebbing's view, there is a chain-like structure to be found in metaphysical analysis, but it does not relate grounded facts to grounds. It relates higher levels of logical construction to lower levels. If we find out that there is a level of non-constructed, basic facts, then we will have discovered that metaphysical analysis is well-founded. But whether it is or not is neither a matter of logic, conceptual analysis, nor of metaphysics, but of empirical discovery.

1 Early Analytic Philosophy and the Assumption That Reality Is Well-Founded

Stebbing was in many respects an archetypal early analytic philosopher. She wrote in detail about the method of analysis (e.g., Stebbing 1932a). She tended to favour anti-idealism. She sought to build a philosophy fit for the twentieth century and beyond around the deliverances of modern physics and mathematics. Yet Stebbing differed from her colleagues Moore, Russell, and Wittgenstein in consistently questioning the well-foundedness of reality.

One key logical atomist argument for well-foundedness rests on the assumption that sense-data play an important role in analysis. Although Stebbing framed her objections to this style of argument as disagreement with Russell, I argue that they plainly also constitute both disagreement with Moore and progress compared to Moore, thus throwing into doubt the prevailing "Moorean" interpretation of Stebbing's metaphysics.[1]

Both Moore and Russell had held, from their very early works (Moore 1899; Russell 1903) that some sort of well-foundedness assumption was necessary in

1 For the interpretation that regards Stebbing as "Moorean," see Milkov (2003, 355, 358); Beaney (2016, 242, 245–246, 248–250, 253–254). For criticism of the "Moorean" interpretation, see Janssen-Lauret (2022a, 2022b, forthcoming).

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order to defeat idealism. They sought to falsify idealism by means of a theory of perception and cognition that sharply separated the mind from its object of judgement, external to that mind (MacBride 2018, 30–39). The two young philosophers aimed to refute both the epistemic idealism of Kant and the idiosyncratic ontological idealism of Bradley by maintaining that, *contra* Kant and Bradley, reality divides into discrete, individually cognisable constituents, and that we know this because our minds can grasp those constituents directly, and our words can name them directly. In subsequent years, Moore and Russell's views evolved away from their early all-words-refer model and towards the more familiar logical atomist model on which a true sentence corresponds to a fact and a false sentence fails to do so. Yet both philosophers remained committed to the position that we can grasp and name at least some constituents of reality directly. They referred to these constituents as "sense-data."

Moore was explicitly open-minded concerning the nature of sense-data. He considered the possibility that they might be the surfaces of objects (Moore 1925, 56), as well as the possibility that they might be mental items. Either way, Moore held, they were pivotal to analysis. Material objects, such as hands, he took to be analysable in terms of the sense-data directly presented to us (Moore 1925, 55–59).

Russell had similarly put forward a strong version of this type of view when he proposed that the process of analysis must terminate in a schedule of sense-data (and universals) to which we have direct cognitive access: "Every proposition which we can understand must be composed wholly of constituents with which we are acquainted" (1911, 117). By holding on tightly to that view, Russell held, we are able to ward off the kind of idealism according to which "we never really, in knowledge, attain to the things we are supposed to be knowing about, but only to the ideas of those things" (1911, 120), because sense-data are indubitable. We cannot be wrong about knowing them. If sense-data are the simples which we know directly and in which analysis terminates, we are on epistemological *terra firma*.

We are all familiar with the problem of epistemic access to unobservable posits of the natural sciences. The same problem occurs for epistemic access to abstract objects such as sets, numbers, logical properties, and functions for those who posit such entities. But even ordinary material objects, animals, plants, rocks, stars, planets, and artefacts, according to Moore, Russell, and Wittgenstein, are not presented to us directly but only via the medium of sense-data. Whether sense-data are themselves the surfaces of such objects or mental

representations of them, belief in material objects such as human bodies or tables then appears to be the result of a risky inference. The classic logical atomist solution, which Stebbing called "Russell's reformulation of Occam's Razor" (Stebbing 1932a, 75; 1933a, 25), was that "logical constructions are to be substituted for inferred entities" (Stebbing 1933a, 25). We cannot observe the table directly. But, Stebbing wrote, "it is nonsense to talk of this table as an 'inferred entity'; hence, they [Russell and his followers] conclude, it must be a logical construct of the given," that is, of sense-data (Stebbing 1933a, 25).

Stebbing took Wittgenstein to share this Russellian assumption but to stray into even more unpalatable conclusions because, she wrote, "Wittgenstein simply takes it for granted that the given is, and could be, nothing but my own direct experience" (Stebbing 1933a, 27). As a result, "every genuine proposition says, and can say, only something about my present or my future experience" (Stebbing 1933a, 27). So Wittgenstein was, Stebbing concluded, forced into "methodological solipsism" (Stebbing 1933a, 27). This conclusion she took to be confirmed by Wittgenstein's statement that "what solipsism means is quite correct, only it cannot be said" (Wittgenstein 1922, 5.62). Stebbing's interpretation of Wittgenstein is controversial. Anscombe's influential reading, for example, presents Wittgenstein as more realist than Stebbing does (Anscombe 1959). I will not assess Stebbing's interpretation of Wittgenstein here; her arguments remain valuable even if she was incorrect to single out the early Wittgenstein as her foil.

While Russell denied solipsism, other minds were, on his view, also firmly in the realm of things which we cannot observe directly. He wrote,

> If a person who knew Bismarck made a judgment about him [..., w]hat this person was acquainted with were certain sense-data which he connected (rightly, we will suppose) with Bismarck's body. His body as a physical object, and still more his mind, were only known as the body and the mind connected with these sense-data. (Russell 1911, 114)

Stebbing, we shall see, took a different view.

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2 Stebbing against Well-Foundedness: The Distinction between Same-Level and Metaphysical Analysis

Like her logical atomist colleagues, Stebbing took one of the main tasks of philosophy to be the clarification of our beliefs and our ordinary-language discourse through philosophical analysis. She further resembled them in positing facts with particulars and universals as constituents, although she also took Whitehead's event ontology seriously as an alternative. But Stebbing's theory of philosophical analysis differed in key respects from those of Russell, Wittgenstein, and Moore. She rejected the idea that analysis terminates in sense-data. She questioned whether we have good reason to believe that some ultimate level of basic facts will be uncovered. She held that what we analyse are sentences, not propositions or judgements. And she made an advance over all of the "founding fathers" by drawing her distinction between same-level and metaphysical analysis and applying it in her metaphysics and philosophy of physics.

Stebbing's original views on analysis entail that we cannot expect reality to simply offer up a basic level of simples or simple facts in which analysis terminates. The existence of simples, or basic facts, is, she argued, not a logical or *a priori* truth. If it is a truth at all, it is one ascertained by inspection of the world. Unlike Russell, Moore, and Wittgenstein, Stebbing distinguished explicitly between metaphysical analysis, which she also called "directional analysis," and same-level analysis.

Same-level analysis analyses a stretch of language in terms of more language. As a result, such analysis remains at the same level of logical construction: the high-level logical construct of language. Stebbing considered linguistic items such as words and sentences to be logical constructs out of tokens that are typographically or phonetically similar or conventionally associated with each other (Stebbing 1935, 9). Analysis of language in terms of more language—such as conceptual analysis, definition, or analytic explication—may well be *a priori*, analytic, or based on synonymy. Metaphysical analysis, according to Stebbing, never is. Metaphysical analysis is concerned not with synonymy or analyticity but with uncovering what schedule of facts, what constituents in what arrangement, there is if a given sentence is true. It requires co-operation from reality. Reality may or may not contain the relevant schedule of facts. The claims "analysis terminates in simples," "there are basic facts," or "analysis terminates in sense-data" are by no means ungainsayable

or logical truths. They can be coherently denied: "that there should be basic facts [is] not logically necessary" (Stebbing 1932a, 80).

Stebbing presented her theory of metaphysical analysis as disagreement with Russell. She framed it as an improvement on his "unfortunate reformulation of Occam's Razor" (Stebbing 1932a, 75)—according to which we should replace inferred entities with logical constructions—and tentatively claimed some affinity with Moore, while noting that Moore did not use the term "metaphysical analysis" and might not agree with her views (Stebbing 1932a, 76, n.1). As a result, some Stebbing scholars have designated her view as "analysis practiced by Moore-Stebbing" (Milkov 2003, 358, 359) or as a "Moorean conception of analysis" (Beaney 2016, 249, 250). It can nevertheless be made apparent, I argue, that Stebbing's metaphysical analysis is distinct from Moore's analysis of propositions and that she was able to solve problems that Moore could not. Although Moore (and Russell) did not intend only to engage in same-level analysis but aimed to uncover "the nature of [...] things" (Moore 1925, 55), Moore, unlike Stebbing, failed to keep metaphysical and same-level analysis sufficiently separate, landing himself in a muddle, which Stebbing managed to swerve.

Moore, in his "Defence of Common Sense," had started off with a strongly anti-idealist message but found himself stymied in the final pages, unable to rule out an idealist analysis of "this is a hand." He took for granted that any such analysis begins with "This is part of the surface of a human hand," a statement that he took to be "undoubtedly a proposition about the sense-datum, which I am seeing" (Moore 1925, 55). Moore considered three analyses: one according to which the hand is a logical construction out of physical sense-data, the surfaces of objects; one according to which it is a construction out of sense-data conceived as mental representations of the material hand; and one according to which it is a construction out of "permanent possibilities of sensation" (Moore 1925, 57)—an idealist analysis à la Mill. Moore argued the first was unable to account for double vision, the second relied on the mysterious relation of "being an appearance of," and then found himself in a kind of aporia, unable to rule out the intuitively unappealing idealist analysis, which he deplored as "paradoxical" (Moore 1925, 59). Although he never explained why he found it paradoxical, I hypothesise that he felt that our common-sense practice of calling hands "material things" (Moore 1925, 42) was at odds with the idealist analysis of hands, since items composed of

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permanent possibilities of sensation would appear to be mental rather than material things.[2]

Stebbing's metaphysical analysis improved upon Moorean analysis by swiftly defanging the apparent paradox. On Stebbing's view, an analysis that has a paradoxical sound to it is problematic only if the analysis in question is a same-level analysis, capturing some form of synonymy or analyticity. Metaphysical analyses may be perfectly viable even though they sound analytically false. A good example is the physical analysis that matter is, at the sub-atomic level, mostly empty space.

Although Stebbing did not press the above point against Moore, she could have done so because she made a comparable move in her rebuttals of idealism in the interpretation of physics. Stebbing's expertise in the philosophy of science had made her well aware of paradoxical-sounding analyses in physics, such as "this solid table is, at the subatomic level, mostly empty space." It would be fallacious to expect a macro-object, or a logical construct, to inherit all the properties of its micro-constituents, or vice versa (Stebbing 1937, 48–54). Just as an acceptable analysis of a dependable physical table may take it to be mostly empty space at the subatomic level, and this analysis does not imply that the table itself is not solid, so could we theoretically conclude that a hand is, at the level of basic facts, made of permanent possibilities of sensation, and this analysis does not imply that the hand itself is not material. There may not be much that can positively be said in favour of the idealist analysis of hands or other material things, and this was indeed the line Stebbing took. But that is a separate issue, to be settled by an investigation of reality. The paradoxical appearance of the idealist analysis is not by itself sufficient reason to dismiss it as a metaphysical analysis.

Stebbing further differed from Moore as well as from Russell and Wittgenstein in dispensing with the central role allocated to sense-data in analysis.

2 An anonymous referee asks whether I read Moore as holding that idealist analyses are incoherent, citing Moore's posthumously published 1928–29 lectures as a counterexample to that reading (Moore 1966, 19), and suggesting that Stebbing is closer to Moore here than I think she is. But I read Moore's description of the idealist analysis as "paradoxical" in (1925) not as ascribing incoherence to the idealist analysis—if that were the case, then Moore would not have found himself unable to rule out the "paradoxical" analysis; he could have dismissed it as illogical—but as connoting that the analysis has an air of analytic falsehood about it. Stebbing, on my interpretation, effectively dispatches the apparent paradox by showing that the idealist analysis need not be false since metaphysical analyses need not be analytically true. Elsewhere, I argue that Stebbing's views on the analysis of physical objects also make an advance over Moore's 1928–29 lectures (Janssen-Lauret forthcoming).

She explicitly rejected Russell's claim that, as she put it, "a *table* is a *class* of appearances" (Stebbing 1933b, 503). We have seen that Moore, too, while not requiring that we view material objects as classes of sense-data, still felt that the process of analysis should have its roots in the type of claim that is "undoubtedly a proposition about the sense-datum" (Moore 1925, 55). But Stebbing's metaphysical method was different: "We must not start from sense-data" (Stebbing 1932a, 72). According to her theory of perception, we are not simply directly presented with sense-data but thrown into a perceptual situation of determinate shades, sounds, smells, *et cetera*. Whenever we start to label individual surfaces or mental states, we have already started the process of generalisation and abstraction in some minor way. Thus, Stebbing wrote, "we must start from the perceptual judgment, made in a given determinate perceptual situation" (Stebbing 1932a, 72) when we engage in metaphysical analysis.

Stebbing was not averse to regarding certain posits as logical constructions, such as classes (Stebbing 1930a, 455), linguistic types (Stebbing 1935), and propositions (Stebbing 1933d, 78). She described going back and forth on the question whether tables and other apparently observable things are logical constructions (Stebbing 1933d, 2). Although she on balance felt that tables count as logical constructions, she objected to the view that they are logical constructions out of sense-data. We may say that tables are immediate referents of discourse about perceptual situations. We then enter into a process of abstraction to analyse what they are made of. They turn out to be made, ultimately, of subatomic particles: the basic facts referred to by sentences about tables are micro-physical ones (Stebbing 1937, 54). Stebbing's views on perception were more akin to those of Whitehead, who, in describing the perceptual situation as an event, sought to overthrow the "bifurcation of nature" into primary and secondary qualities and into mind and body, than to those of Moore.

Stebbing's views on perception had grown out of her engagement with physics and its philosophy. In a relatively early paper, she defended a position she called "realism," according to which both philosophy and the natural sciences start from "perceptual science" (Stebbing 1929, 147), comprising statements such as "I am perceiving a piece of paper," "the piece of paper was here before I saw it," and "others have seen this piece of paper, too." Stebbing's perceptual science explicitly encompasses physics as well as philosophy. It takes it as a given that perceptual objects have a duration and that other minds exist and can perceive the same objects. That other minds are not things to be

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known only by means of risky inference or logical construction, she appears to have taken as a clear and basic feature of the scientific method. Later, she was to criticise Eddington, for example, for thinking that "the inquiry concerning the nature of an other mind (called 'Mr. X') 'must take place in the domain of my own consciousness' (Eddington 1928, 268)"; Stebbing countered, "The difficulty is that Mr. X—indeed an army of Mr. X's—must be assumed if physics is to be possible" (Stebbing 1937, 109). Here we see her invoking her 1929 proposal of "perceptual science," which has reliance on the observations of other observers running all the way through the scientific method like a stick of rock.

In the same vein, we may read Stebbing's rebuttal of the methological solipsism she attributed to Wittgenstein as harkening back to her (1929) perceptual science. Stebbing's reply to the methodological solipsist is brief: "I have the best of grounds for denying solipsism, namely, that I *know* it to be false. You, who are listening to me, and enable me to speak in the plural *also* know it to be false" (Stebbing 1933a, 27). Although at this point she included a footnote to Moore, who, unlike Russell and Wittgenstein, also took the assumption of other minds to be basic, this position was really original with Stebbing herself. And, I argue, it derived from her consideration of the scientific method rather than from a "Moorean" sense that the denial of other minds is paradoxical. In 1929, she had relied on the principle "Other people besides myself have seen that piece of blotting paper" (Stebbing 1929, 1) as part of perceptual science, a pragmatically necessary assumption for progress in both physics and philosophy.

3 Stebbing's Metaphysics and the Question of Well-Foundedness: Metaphysical Analysis Post-1934

We have seen that in Stebbing's estimation, the question whether reality is well-founded cannot be settled *a priori* because neither well-foundedness nor its negation is a logical or conceptual truth.[3] Statements such as "there are simples" or "there are basic facts" can be coherently affirmed or denied. So can "analysis has no stopping point." Same-level analysis, such as conceptual analysis, or analysis within a completely conventional or postulational system,

3 Chris Daly points out that statements about mathematical objects might provide a challenge to the principle at work here. Stebbing's reply would have invoked a version of the no-class theory she endorsed, according to which sets and numbers are logical constructions (Stebbing 1930a, 158).

will not help us answer the question of the well-foundedness of reality. How, then, did Stebbing propose to answer it? Although the passages in which Stebbing indicates what her answer would look like are compressed, I believe that they contain promising material to inform the present-day debate.

In brief, Stebbing's alternative answer was that if there are simples, they have to be found in the world by means of the method of metaphysical analysis. Stebbing's metaphysical method was somewhat naturalistic. She took physics and philosophy to share the same starting point of "perceptual science." She looked towards physics, rather than sense-data, to find out what hands and tables were made of. Where Moore asserted that analysis begins with what is "undoubtedly a proposition about the sense-datum" (Moore 1925, 55), and Russell, though interested in the philosophy of physics, contended that a table is a set of sense-data, Stebbing's examples of analysis tended towards the physical: water is made of hydrogen and oxygen (Stebbing 1932a, 67), matter is ultimately made of sub-atomic particles (Stebbing 1937, 48–51). Instead of the analyses which she attributed to Russell and Wittgenstein, respectively—taking a table to be "a class of appearances" (Stebbing 1933b, 502) or "an experience of mine" (Stebbing 1933a, 28)—Stebbing held that tables ultimately consisted of quarks and leptons (Stebbing 1937, 54): some quarks are arranged into protons, combine with electrons to form atoms, atoms of various sorts combine to form molecules, arranged into cells, arranged into cellulose fibres, arranged into planks, in turn arranged table-wise. The quarks' proton-wise arrangements here play the role of basic facts. Evidently, their playing this role is not a conceptual truth but an empirical discovery. She also regularly drew upon examples of the analysis of socially constructed entities in terms of natural entities: "The action of the Council is a logical construction out of a set of facts each of which is a fact about one individual member" (Stebbing 1930a, 504).

Like Carnap and Russell, Stebbing felt that analytic philosophy had a role to play in spelling out the implications of the increasingly structural theories of the new Einsteinian physics, which yielded "a constructed system stated in terms of imperceptibles, the system being such that it permits, under certain conditions, of interpretation by reference to perceptual elements" (Stebbing 1933d, 9). Stebbing expressed sympathy for the tradition of Pearson, Mach, and Kirchoff, also embraced by Carnap, according to which "science does not *explain* but *describe*" (Stebbing 1930a, 392). Yet she sought to refine some of their rather crude pronouncements, since "a *complete* description of natural motions [as Kirchoff proposed] is impossible, and if it were not

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impossible it would be useless" (Stebbing 1930a, 393). Instead, a fruitful scientific theory takes the form of a "constructive description," which provides fruitful abstractions and generalisations by "attending to certain properties of what there is in Nature, by constructing hypothetical entities (i.e. constructs) whose function is to aid in the correlation of what is sensibly observed, and by using whatever mathematical methods may serve to develop the correlation" (Stebbing 1937, 91).

Historians usually say that Stebbing abandoned metaphysical analysis, either after (1933d) (Chapman 2013, 94) or at least by (1939) (Beaney 2016, 253). On my interpretation, Stebbing did not abandon metaphysical analysis and, in fact, continued to apply it in her philosophy of physics.[4] It is true that in (1939) and in (1942), she disavowed the exact account of metaphysical analysis that she had given in (1932a). But, read closely, these disavowals are of the Russell-Moore well-foundedness assumption that analysis terminates in sense-data, not of the distinction between metaphysical and same-level analysis. Stebbing wrote, "I was protesting against the view that there is any problem of justifying inferences from sense-data to perceptual objects" (Stebbing 1939, 73). What she had abandoned by then was her (1932a) hope that Moore's project might coincide with hers: "I did not then clearly see that Moore's discussion was also entangled with epistemological problems" (Stebbing 1939, 73), as Russell's had been. In her (1942) retrospective on Moore, Stebbing again made clear that Moorean analysis was, in her view, stymied by the well-foundedness assumption: "Moore certainly has suggested that the analysis must terminate in sets of propositions about sense-data [...] There seems to me to be no good reason for asserting that there are such" (Stebbing 1942, 527). Stebbing did, at times, express the worry whether metaphysical analysis without the well-foundedness assumption could be on sufficiently solid ground. But that did not stop her from apparently applying her method of metaphysical analysis in her philosophy of physics. Although she did not invoke it by name there, the analysis she deployed must be metaphysical analysis since it applies to statements that appear analytically false at the level of ordinary language.

Physicists such as Eddington had argued that the modern theory of sub-atomic particles meant that matter could no longer be viewed as solid. Stebbing argued that this inference relied on the fallacious assumption that predicates

4 For a fuller account of Stebbing's applications of directional analysis in her philosophy of physics, see Janssen-Lauret (2022a, 32–44; 2022b; forthcoming).

that apply to macro-physical objects also apply to their micro-physical components, so that if a macro-object is hard and solid, all its constituents are hard and solid. But such a principle, though it may have an analytically true sound to it—just like Moore's assumption that if a hand is material, its constituent parts should be material—may be falsified in the case of physical and metaphysical analysis. Indeed, Stebbing wrote, it is so falsified because "no concepts drawn from the level of common-sense thinking are appropriate to sub-atomic, i.e. microphysical, phenomena" (Stebbing 1937, 51). Instead, "it would be more appropriate to say that the modern physicist no longer believes that the table consists of solid atomic balls, than to say that 'the table no longer possesses solid reality' " (Stebbing 1937, 54).

Stebbing's opposition to Russell's contention that "every proposition which we can understand must be composed wholly of constituents with which we are acquainted" (Russell 1911, 117) may also have derived from her understanding of physics. If physical simples are quarks or electrons—we know that physical objects can be at least as small as this—then some physical simples resist knowledge by acquaintance. As a matter of physics, we are unable to observe electrons directly. Electrons are smaller than the wavelength of visible light (Stebbing 1937, 181). So Stebbing knew that there were some simples, arguably termini of metaphysical analysis as practiced in physics, with which we cannot possibly be acquainted. This fact further confirmed her position that analysis should not be expected to terminate in simples, which are the objects of acquaintance.

Though naturalistically oriented, Stebbing's system was more metaphysical than that of Carnap, whose *Aufbau* Stebbing admired but critiqued for engaging only in same-level analysis (Stebbing 1933a, 1933d), or the later self-professed naturalism of Quine. Quine would have agreed with Stebbing's dictum that "the metaphysician is not concerned to discover any new facts; he does not add to the sum-total of human knowledge in the way in which the natural scientist or the historian does" (Stebbing 1932a, 65). The Quinean naturalist philosopher famously builds her philosophy around the deliverances of the sciences instead of seeking to build a prior metaphysical foundation for them, and so, too, for the Stebbingesque naturalist. But Stebbing made clear that she regarded metaphysics as "a distinctive branch of philosophy" (Stebbing 1932a, 65) with its own methods, though not with its own bespoke metaphysical relations or facts.

Stebbing set apart her own chosen method, the method of metaphysical analysis, from other, more traditional methods in metaphysics, such as the

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deductive method of Spinoza and McTaggart, which rests upon axioms (Stebbing 1932a, 66–67), and Aristotle's methods (Stebbing 1930a, 432–434). Her own method of metaphysical analysis implies a metaphysics of levels. Her other term for metaphysical analysis, "directional analysis," indicates that the process tends towards ever greater simplicity. Its goal is to "determine the elements and the mode of combination of those elements to which reference is made when any given true assertion is made" (Stebbing 1932a, 79). Stebbing's metaphysics of levels can be precisely characterised; the language of "levels" is not metaphorical. The lowest level is the level of simples, if there are any, combining into basic facts. Higher levels represent increasing amounts of logical constructedness. Same-level analysis, as we have seen, connects stretches of language—high-level logical constructions out of language-tokens—to more language, remaining at the same level. So, although same-level analysis may be said to have a "direction" in the sense that, for example, the right-hand side of a definition elucidates the left-hand side, it is not directional analysis in Stebbing's precise, technical sense of descending down the levels in search of greater simplicity.

Stebbing's range of ontological categories was also greater than Quine's. She had long accepted the existence of particulars and universals as basic, though admitting that Whitehead's event ontology was a worthy alternative against which she could not offer any strong arguments (1925, 315–316). Stebbing posited facts, with universals and particulars as constituents, writing, "A configuration of elements is what is usually called *a fact*. To the *configuration* is due the unity of the fact; to the *elements* it is due that there is something to be configured" (Stebbing 1932a, 80, her italics).

4 Analysis of Sentences vs. Analysis of Facts: Comparison with Recent Debates

Stebbing's metaphysics bears some resemblance to truth-maker theory, to which it has been compared (Egerton 2021). One obvious respect of resemblance between truth-making and metaphysical analysis is that both Stebbing and most truth-maker theorists posit facts whose constituents are particulars and what Stebbing sometimes called "characteristics," namely properties or relations (Stebbing 1933d). The other respect of resemblance is Stebbing's contention that what philosophers analyse are not concepts or things, but sentences. Unlike many present-day truth-maker theorists, Stebbing also denied

that we analyse propositions in metaphysical analysis. Whether the truth-bearers are sentences or propositions, the view that analysis has a truth-bearer as a point of departure and ends with facts remains a striking commonality between metaphysical analysis and truth-maker theory. Yet I will argue that the surface-level similarity is, to an extent, deceptive.

The proposal that we analyse sentences, as opposed to concepts or propositions, is one that Stebbing presented as a difference between her and Moore. She wrote,

> I prefer to use somewhat different language from that used by Moore [...] Where he speaks of "knowing what a proposition means, in the sense of being able to give a correct analysis of its meaning" I prefer to speak of "knowing the analysis of a sentence" [...] I believe that what we analyse are expressions, of which sentences are one kind; and that when we analyse a sentence expressing a proposition what we obtain is not another proposition but another expression. (Stebbing 1933a, 9)

We saw above that Russell, too, had spoken of "propositions," but he used "proposition" in a sense in which the contemporary debate uses "fact." Stebbing, by contrast, wrote, "I think that a proposition is a logical construction out of a set of facts in which someone is using a sentence to express what he is truly or falsely judging." As her propositions were not *sui generis* meaning-entities residing in an abstract third realm, her use of the term is not precisely Fregean, but it is clear that she did not use "proposition" in its Russellian sense.

Stebbing, then, bypassed propositions as meaning-entities in her metaphysical analysis. She viewed metaphysical analysis as crucially involving sentences and other linguistic expressions on the one hand and facts on the other, writing, "Metaphysics is a systematic study concerned to show what is the structure of the facts in the world to which reference is made, with varying degrees of indirectness, whenever a true statement is made" by means of a sentence (Stebbing 1932a, 65). Stebbing appears to have had in mind that metaphysicians analyse sentences as used on a given occasion rather than abstract sentence types. She noted frequently that sentences of the same type may have different meanings on different occasions of use (Stebbing 1930a, 149).

Although Stebbing's system resembles modern truth-making theory in positing the ontological categories of fact, particular, and universal, there are

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also striking differences. One is Stebbing's lack of reliance on modality or intensionality. In taking truth-bearers to be sentences rather than propositions, as well as in being devoid of any assumption of necessitation between sentence and fact, Stebbing's position resembles the views of Quine, Tarski, Goodman, and other mid-analytic extensionalists more than present-day truth-making theory. Note that the Stebbing quotation in the previous paragraph is entirely non-modal. It contains no "must" or "ought" or "necessarily." The same is true of her formulation of the aim of metaphysical analysis, to "determine the elements and the mode of combination of those elements to which reference is made when any given true assertion is made" (Stebbing 1932a, 79).

These extensional formulations were neither a coincidence nor the result of an oversight on Stebbing's part. Apart from potential Humean scruples about the necessary connection between particular and universal in a fact (MacBride and Janssen-Lauret 2022, 83–84), Stebbing's position was largely compatible with Quinean-Tarskian extensionalism.[5] She consistently disavowed the ascription of metaphysical necessity to the world (Stebbing 1930a, 175–176, 265–266, 433). Stebbing countenanced "logical necessity" (Stebbing 1930b, 285) and the necessity of analytic truths (Stebbing 1933c, 193), but not, apparently, metaphysical necessity. She did not regard causation as necessitation, writing instead, in her chapter titled "Causality," "The question of necessity does not arise for the practical agent and cannot arise for the scientific investigator until he has generalized from the particular instances so as to obtain the form *whenever X, then E*" (Stebbing 1930a, 265).

The truth-maker theory of the late twentieth and early twenty-first century, in addition to positing propositions as truth-bearers and facts as truth-makers, also often posits a peculiarly metaphysical relation of truth-making. According to these accounts, truth-making is necessitation (Armstrong 2004, 5), the relation in virtue of which something is true (Armstrong 1989, 88), or a link between a fact and the essence of a truth-bearer (Lowe 2006, 203–210). Stebbing, by contrast, declined altogether to posit essences or bespoke metaphysical facts and relations: "metaphysics is not concerned with a distinctive region of fact" (Stebbing 1932a, 66). Stebbing's thesis that metaphysics does not have its own distinctive subject matter in conjunction with her moderate naturalism yielded the view that a modern, scientifically informed philosophy develops in tandem with modern science to dispense with notions of determinative

5 I have argued previously that Stebbing was a moderate extensionalist (Janssen-Lauret 2022a, 27–28). Her view was less radical than Quine and Tarski's, but she disavowed abstract intentions.

explanation, necessitation, and essence.[6] Stebbing wrote, "Modern theories of organic evolution have combined with modern theories of mathematics to destroy the basis of the Aristotelian conception of essence" (Stebbing 1930a, 433).

Although an account of truth-making as entailment (cf. MacBride 2013, sec. 1.1), which Stebbing regarded as a primitive logical relation (Stebbing 1930a, 222), might in principle have been open to her, Stebbing did not take that path. Instead, she affirmed that the entailment relation runs both from truth-bearer to analysis and vice versa: "If $\pi_1, \pi_2, \ldots \pi_n$ is the analysis of p, then p entails and is entailed by $\pi_1, \pi_2, \ldots \pi_n$" (Stebbing 1932a, 85).

Yet Stebbing's text pulls in two different directions concerning the question of what metaphysicians analyse. In addition to speaking of metaphysical analysis as a relation between a sentence and an array of basic facts, she at times also described it simply as an analysis of facts. For example, "At present I shall use this expression ['proposition'], but later I shall inquire whether the analysandum may be regarded as a fact" (Stebbing 1932a, 77) and "I think that metaphysics is primarily concerned with the analysis of facts; it is not concerned with the analysis of things, though the special natural sciences may be so concerned" (Stebbing 1932b, 310).

The twenty-first-century reader might be tempted to draw a different comparison here: that Stebbing's metaphysical analysis is not like truth-making but like grounding. Grounding is often taken to be a relation between a fact and another fact or facts, which is asymmetric and transitive, and which can form a chain-like structure with p being grounded in q being grounded in r and s. Whether it is well-founded is a subject of debate (e.g., Dixon 2016). The truth-making debate is largely unconcerned with questions of well-foundedness, chain-like structures, and their logical properties, such as asymmetry or transitivity. Truth-making need not be a one-one relation, with one truth-making fact per proposition; logically molecular propositions might be made true by sets or collections of atomic facts instead of by logically molecular facts. Nevertheless, the truth-making relation is not generally thought to have a chain-like structure (Fine 2001, 25). Metaphysical analysis, by contrast, does, and Stebbing explicitly commented on it.

There are some passages in Stebbing's work that, in isolation, appear to suggest that the question of well-foundedness might be fruitfully addressed

6 Hence also Stebbing's statement that "the metaphysician is not concerned to discover any new facts" (Stebbing 1932a, 65); see section 3 above. I am grateful to an anonymous referee for encouraging me to discuss this connection more.

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by reading metaphysical analysis as a type of, or analogue of, grounding and bringing Stebbing's answers under the grounding umbrella. But I will argue that an interpretation of Stebbing in grounding terms cannot be maintained. Stebbing consistently argued against many of the metaphysical tools in the grounding theorist's arsenal: metaphysical determination, priority, essence, metaphysical "why"-questions in science, metaphysical ultimacy, and the metaphysical distinction between appearance and reality. What's more, several prominent grounding theorists compare analysis unfavourably to grounding, but I will show that from Stebbing's text we can extract a promising argument in favour of analysis over grounding.

At first, the case in favour of reading Stebbing's metaphysical analysis as a grounding-analogue may seem strong based on certain passages. In the following, she appears to attribute a chain-like structure to an array of non-basic and basic facts:

> A fact F is based upon a fact F' when F cannot be unless F' is. If F is based upon F', then F contains a configured element F'. Since a simple fact contains no configured elements, it cannot itself be based upon any other fact. (Stebbing 1932a, 80)

> The immediate reference of a proposition is never a basic fact, but it is in conformity with usage to say that a proposition asserts a fact, and if the proposition be true there is an ultimate reference to basic facts. We cannot tell by simple inspection whether a proposition is true or false, but we can determine the immediate reference of any proposition. A proposition is an assertion; an assertion entails a thinker, but the immediate reference of a true proposition does not depend upon its being asserted. Consequently, we must admit that there are non-basic facts. But non-basic facts are facts of a different kind from basic facts. (Stebbing 1932a, 81)

Elsewhere, she commented on the logical properties of the chain relation, calling it "asymmetrical and transitive," properties that are also widely ascribed to the relation of grounding:

> I am in the habit of describing the analysis involved in metaphysical inquiry *directional* in order to contrast it with other forms of analysis, which may be circular. To say that the analysis of F is *directional* is to say that if F be analysed into a, b, c, then a, b, c,

> are on a lower level than F; and if a be analysed into a_1, a_2, then a_1, a_2 are on a lower level than a. The relation of *being on a lower level than* is clearly asymmetrical and transitive. To say that a is on a lower level than F is to say that a is in some sense *simpler than F*. (Stebbing 1932b, 311, n.4)

She added that while metaphysical analysis is assumed to be well-founded, assumed to terminate in simples, "there is a tendency to *assume* [i.e., without argument] that an ultimate element is an absolutely simple element" (Stebbing 1932a, 89), but this, again, is an assumption that can be false because it is not logically or conceptually necessary. Perhaps simples can be discovered or encountered in the world. Stebbing found it "plausible" that "an *absolutely specific* shade of colour, or taste, or sound, may be simple in the required sense" (Stebbing 1932a, 91). Nevertheless, she wrote, that suggestion remains logically contingent, beholden to what reality is actually like: "To assert that a basic fact is an absolutely specific fact is to make a significant assertion about the constitution of the world. It is not to make an assertion about synonymous expressions. It may be false" (Stebbing 1932a, 89). She therefore worried that it is uncertain whether, in circumstances where the field is dominated by the well-foundedness assumption, we can do metaphysics at all. "Metaphysics awaits its Galileo" (Stebbing 1932a, 93).

5 Analysis of Facts in What Sense? Stebbing against Grounding

A central claim of many versions of contemporary grounding theory is that we certainly can do metaphysics, appealing to bespoke metaphysical grounding or determination relations. Some are also explicit that this method is to be preferred to analysis (Fine 2001; Berker 2018). But Stebbing would not have awarded the founder of grounding the title of "metaphysical Galileo." We have seen that she opposed positing specifically metaphysical relations. She supported her claim that "metaphysics is not concerned with a distinctive region of fact" (Stebbing 1932a, 66) with detailed arguments against the existence of distinctively metaphysical determination relations.

Canonical statements of the grounding project include "there is a primitive metaphysical concept of reality" (Fine 2001, 1). Such statements also explicitly trade on the contrast between appearance and reality (Fine 2012, 41). Stebbing

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objected that such metaphysical claims can neither be empirically supported nor are they generally supported by sound metaphysical argument:

> The phrase "ultimate nature of reality" implies that reality has a nature that is not apparent. "Ultimate" cannot here be so interpreted as to signify that which could be discovered by analysis or by experimental observation. [Metaphysicians assume] the opposition of Reality to Appearance. It is important to ask what is the nature of this opposition.
>
> Consider the opposition of a chemical compound to its constituents. […] Only a very muddled chemist could suppose that hydrogen is more ultimate than water in any sense other than "chemically more simple." The case is quite otherwise, however, with the opposition of the ultimate nature of reality to its apparent nature. This distinction is not yielded by experimental observation; it is not *yielded* at all. On the contrary the philosopher who accepts the distinction *starts* from the ultimate. (Stebbing 1932a, 67)

Advocates of grounding would counter that they do have sound metaphysical argument to support their views, for example, via appeal to a "constitutive conception of essence" (Fine 2012, 71), a conception going back to the Aristotelian roots of essentialism, free from the modern disease of conflating an essence with a necessary property. Stebbing, who knew Aristotle's text very well, cannot be accused of conflating essence and necessity. Though correctly describing Aristotelian essence in detail, Stebbing made clear that it did not meet her standards of intelligibility.

> Aristotle's notion of essence is difficult to understand. He nowhere clearly explains it, but seems to take "essence" as a technical term to be left undefined and by means of which he defines those predicables that are to be contrasted with it. (Stebbing 1930a, 429–430)

Stebbing's standards for intelligibility in metaphysical explanation were exacting. This is evident, for example, from her reflections on the use of mereological composition terminology:

> It makes sense to say that lemonade is composed of lemon-juice, water, and sugar. […] It makes sense to say that water is composed

> of oxygen and hydrogen, although this is a different usage of "composed of" from the usage in the statement about lemonade. […] But what meaning can be assigned to "the ocean is composed of water?" […] To this, it seems to me, the correct answer is that the question involves a misuse, or at best a wildly Pickwickian use, of "composed of." (Stebbing 1937, 87–88)

Her intelligibility constraints appear to rule out the "classical mereological relation" (Wilson 2014, 539; Berker 2018, 763) discussed in the modern grounding debate, which allows for statements such as "the ocean is composed of water."

Along similar lines, Stebbing argued that the metaphysical use of the vocabulary of "priority," "determination," "ultimacy," "appearance," and the like goes beyond our ordinary-language use of cognate terms and strays into the realm of misuse. We have seen that she was willing to admit logical necessity, though not specifically metaphysical necessity. Stebbing was careful to ward off the intrusion of metaphysics into her logic. Concerning the proposal that axioms may be defined in terms of logical priority, she objected, "But logical priority is not absolute. The notion of logical priority is obscure. Its discussion has been encumbered with difficult and dubious metaphysical assumptions" (Stebbing 1930a, 175). She went on to argue that priority could perhaps be defined in terms of simplicity, but that as "simplicity is also a relative notion" (Stebbing 1930a, 175), not an absolute one, the problem is not thereby solved.

Stebbing was happy with the use of the word "determination" in its mental or epistemological sense, as, for example, in "the determination by experiment of those properties of phenomena that vary quantitatively" (Stebbing 1930a, 313). But she warned that in slipping into using "determines" in a metaphysical sense, we may unwittingly slide into running those two senses together: "The question of one-one [causal] determination belongs to the retrospective attitude; it concerns knowledge, not action. […] The statement of a causal uniformity is a generalization; consequently, it involves abstraction" (Stebbing 1930a, 264).

Stebbing's arguments here found an echo in Carnap, who, one year later, noted that "prior," etymologically speaking, means "before." In its metaphysical use, by contrast, "it is not supposed to mean the temporally prior any more, but the prior in some other, specifically metaphysical, respect" (Carnap

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1931, 225). Carnap, of course, drew much stronger, globally anti-metaphysical conclusions from this lack of intelligibility.[7] Soon afterwards, Stebbing wrote,

> The Logical Positivists, including Wittgenstein, agree in rejecting certain traditional, and still not uncommon views, concerning the nature of philosophy. […] With this rejection I also agree. The views rejected are those which hold that philosophy is concerned with the "ultimate nature of reality." But in this phrase "ultimate" stands for nothing. (Stebbing 1933a, 5)

Stebbing, of course, did not derive an anti-metaphysical conclusion from her rejection of such traditional metaphysics. We have seen that she believed in a specifically metaphysical method and affirmed ontological commitments to particulars, universals, facts, and physical objects. Yet Stebbing's reservations about the vocabulary of "priority," "determination," "ultimacy," and "essence" were not of the nature of a merely sceptical challenge, holding out for this vocabulary to prove its usefulness, as modern critics of grounding often do (Wilson 2014; Koslicki 2020), nor even the stronger kind of scepticism that largely questions the intelligibility of "grounding" and its associated vocabulary (Daly 2012). She disavowed necessity, essence, and classical mereology. She took her arguments to license a fully-fledged "rejection" of metaphysical systems, which assumed that such vocabulary referred to bespoke metaphysical relations. Her anti-grounding conclusions were stronger than those of Wilson and Koslicki, who admit determination relations but no overarching grounding relation, or even Daly, who proposes that grounding claims might be "cases of restricted necessities" (Daly 2012, 98).[8]

Stebbing would also have opposed monism, both substance monism and priority monism, as a solution to the problem of well-foundedness. Substance monists maintain that there is ("ultimately") only one thing. But in Stebbing's view, " 'ultimate' stands for nothing" (Stebbing 1933a, 5) in its metaphysical use, and the assumption that there is only one thing runs afoul of the method of perceptual science. To construct a physical theory requires a plurality of observers and theorists, and the same is true of philosophy (Stebbing 1929, 147; 1937, 108–109). Theorising is impossible without the assumption that multiple perceiving minds can perceive the same, mind-external objects. Pluralism is, of

7 For a Carnapian case against grounding, see MacBride and Janssen-Lauret (forthcoming).

8 Daly tells me (personal communication) that he now considers that wording slightly misleading; he meant that grounding talk can be dispensed with and replaced with claims of restricted necessity. He considers his own position to be close to Stebbing's "rejectionism" about grounding.

course, not a logical truth. We can, without contradiction, say that there is only one thing. But, Stebbing would have said, I cannot intelligibly maintain that there is only one thing when I do physics or philosophy of physics. I cannot do philosophy and coherently maintain that there is only one thing if I adhere to even a moderate philosophical naturalism. As she objected to Wittgenstein and the early Carnap, *we* (multiple persons) know that the assumption that there is only one thing is false. It is falsified when I draw upon physical knowledge or when I interact with other people. "Theoretical physics has developed by the continual modification of common-sense views through a stage of what might be called perceptual science [...] unless perceptual science is true theoretical physics cannot be true" (Stebbing 1929, 148). Perceptual science includes the assumption of multiple minds.

Monism failed, as far as Stebbing was concerned, both because of monists' denial of the naturalistically necessary assumption of the ultimate existence of multiple perceiving minds and because of their persistent reliance on "Aristotle's notion of *priority in nature*" (Schaffer 2018, his italics) to argue that the whole is metaphysically prior to, or more ultimate than, the parts. Stebbing maintained that Aristotelian metaphysics of this sort was of dubious intelligibility and had, in any case, been obviated by modern science (Stebbing 1930a, 433).

Last, I will extract from Stebbing's text a response she could, and likely would, have made against the charge that analysis is merely linguistic and therefore inferior to grounding. Fine, for example, writes that when we analyse "the couple Jack and Jill is married" as "Jill is married to Jack," it is "the point of the reduction to show that couples are a 'logical fiction' and hence not really existent," but objects that such "reduction is a *semantical* matter" (Fine 2001, 9, his italics). And Berker objects that in metaphysical disputes, proponents of a certain view "disagree with their opponents—and with each other—over *substantive* matters, not over *linguistic* or *conceptual* matters" (Berker 2018, 739, his italics).

Stebbing would have considered the Fine-Berker objection to analysis to rest on a clear mistake, the mistake of conflating same-level analysis with metaphysical analysis. Same-level analysis is linguistic or conceptual, explicating language in terms of more language. When we engage in metaphysical analysis, by contrast, we "determine the elements and the mode of combination of those elements to which reference is made when any given true assertion is made" (Stebbing 1932a, 79)—where "determine," as usual with

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Stebbing, is used in the epistemic rather than the metaphysical sense of the word.

Stebbing's metaphysical analysis is neither linguistic nor conceptual. It gives a full account of the basic facts, the constituents of the world, and their arrangement, which are there if the sentence is true. Stebbing, like Fine, disapproved of the implications of calling logical constructions "fictions" (Stebbing 1933b, 502) because couples, for example, are not fictitious. But, unlike Fine, Stebbing held that the actions and properties of couples can always be satisfactorily explained in terms that mention only the individual members of the couple. So it is appropriate, by her lights, to regard a couple as a construct. We do for couples as we do for councils:

> We may say that a College, or the Council of a College, or a Committee, or a Nation, have acted in a certain way. Thus, for example, we may say, "The Council have elected A as chairman." This statement says something about each member of the Council, but it does not say of each member that he elected A. But a set of statements could be found, jointly equivalent to the original statement, which would be each a statement about one individual member. The action of the Council is a logical construction. (Stebbing 1930a, 504)

Stebbing went further and argued that metaphysical analysis is, in fact, the only way to step outside a cycle of same-level definitions, a merely postulational system, and formulate metaphysical hypotheses about what demands our claims really make on reality. To start with a specification of a constitutive essence, a question of the form "What is it to be a so-and-so?" she argued, would be useless because it traps us in a postulational cycle. Metaphysicians who attempt to begin with the specification of constitutive natures have thereby gained no knowledge of whether anything exists that really has that nature. While they intended to investigate reality, in practice they remain stuck engaging in same-level analysis instead of getting down to the proper business of metaphysics, metaphysical analysis. Without metaphysical analysis, they have no way to get at reality.

> The point I wish to emphasize is that it is a grave defect in metaphysical method to begin the investigation of problems by asking: What is it to be a so-and-so? For example: What is it to be a material thing? What is it to be a cause? The only possible form of

> answer to such a question is a definition, which leads us nowhere. We must begin with commonsense facts, such as *I see this candle*, or *This blow on his head killed this man*, or *Her remarks made him angry*. It is useless first to define "material thing" or "cause" and then to ask whether the terms so defined are exemplified in the world. (Stebbing 1932a, 74)

In summary, Stebbing's trenchant objections to many of the pivotal arguments and machinery of grounding mean that a grounding interpretation of Stebbing cannot be upheld. How, then, are we to account for Stebbing's passages quoted above stating that "the analysandum may be regarded as a fact" (Stebbing 1932a, 77), that "we must admit that there are non-basic facts [...] non-basic facts are facts of a different kind from basic facts" (Stebbing 1932a, 81), and that "the relation of *being on a lower level than* is clearly asymmetrical and transitive" (Stebbing 1932b, 311, n.4)? These seem to present metaphysical analysis as a chain-relation that links a fact to another fact or facts, linked to another fact or facts, and so forth. I suggest that there are two possible readings that make sense of the Stebbing passages that I quoted at the end of section 4.

According to the first possible reading, Stebbing meant that there really are non-basic facts, involving properties and relations distinct from those involved in basic facts, but the properties and relations in question are physical (or biological, or mental), not metaphysical. In Stebbing's claim that "a fact F is based upon a fact F' when F cannot be unless F' is" (Stebbing 1932a, 80), the "cannot" must be read as expressing not metaphysical necessity but either logical necessity or a restricted physical necessity. Textual evidence clearly revealed that Stebbing did not believe in higher-level facts formed from special metaphysical relations like necessitation, constitution, or classical mereology. Yet this does not exclude the possibility of some higher levels of facts consisting of lower-level ones standing in physical relations, known to us as the result of empirical discovery. Two up-quarks and a down-quark combine into a proton, which is orbited by an electron to form a hydrogen atom; the hydrogen atom's electron combines with another atom's electron into a cloud to create a chemical bond that holds together a molecule; macromolecules combine into DNA strands, *et cetera*, all composing physical or biological facts.

On this reading, it is unproblematically and literally true both that there are certain non-basic facts and that there are no distinctively metaphysical relations or distinctively metaphysical facts. Macro-facts are formed out of

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basic facts plus physical properties and relations, and perhaps specifically chemical, biological, physiological, or psychological properties and relations. While this interpretation makes ready sense of much of Stebbing's text, it is not obvious that it fully accounts for the chain-like structure that Stebbing attributes to "being on a lower level."

According to the second possible reading, all of the levels except that of basic facts are strictly speaking levels of logical construction, and some of Stebbing's discourse about non-basic facts must be read as a mere façon de parler, a dispensable shorthand to be explicated in terms of incomplete symbol theory. On this interpretation, Stebbing's claim that "we must admit that there are non-basic facts. But non-basic facts are facts of a different kind from basic facts" (Stebbing 1932a, 81) constitutes an oblique way of expressing that we must admit the "non-basic-fact" manner of speaking, even though reality only contains basic facts. As Stebbing put the point elsewhere, "To say that the table is a logical fiction (or construction) is not to say that the table is a fictitious, or an imaginary, object; it is rather to deny that, in any ordinary sense, it is an object at all" (Stebbing 1930a, 502). When Stebbing wrote, "The relation of *being on a lower level than* is clearly asymmetrical and transitive. To say that *a* is on a lower level than *F* is to say that *a* is in some sense *simpler than F*" (Stebbing 1932b, 311, n.4), she meant that the relational predicate "being on a lower level than" does not really stand for a relation because it is always flanked on at least one side by an incomplete symbol, which disappears upon analysis.

There is some textual evidence in favour of this second reading of Stebbing. She wrote, for example,

> We may perhaps say that "S" in a given usage is an incomplete symbol when "S" occurs in an expression expressing a proposition and "S" is neither a name nor a descriptive phrase referring to a particular which is a constituent of the proposition through some property belonging to a particular. (Stebbing 1930a, 156)

Stebbing made explicit that, concerning discourse about linguistic types, propositions, sets or classes, numbers, and mathematical points and lines, she took the line that these are logical constructs that disappear upon analysis. At times, she also suggested taking this line concerning macro-physical objects:

> It does not make sense to say that a logical construction can be substituted for a persistent substantival object, although it is sense

> to say that a table is not a persistent substantival object, and that every statement about this table can be finally translated into a set of sentences in which the word "table" does not occur. (Stebbing 1933d, 23)

Another advantage of the latter interpretation is that Stebbing's theory, on this interpretation, bridges the gap between truth-making, which relates a sentence to a fact, and the kind of metaphysical explanation that relates facts to a further fact or facts and forms a chain-like structure. The contemporary debate assumes that truth-making cannot form chains because sentences or propositions, on the one hand, and facts, on the other, are not sufficiently alike in kind. But if the chain-like structure is a feature specifically of the relation of "being on a higher level of logical construction," then since sentences (or propositions) and non-basic facts are both logical constructs, they are sufficiently alike to feature in the same role in the chain-like structure.

6 Conclusion

Stebbing's positive proposal concerning the question of well-foundedness is one that combines her own, *sui generis* kind of metaphysical analysis of perceptual and other ordinary facts with a certain kind of naturalism according to which questions about the structure of reality need to be approached by a divide-and-conquer method assigning different sub-questions to different branches of science and philosophy (Stebbing 1943). Analytic or a priori methods will not settle the question whether reality is well-founded. Modern physics proves incompatible, in different ways, with the atomisms of Russell, of Moore, and of Wittgenstein and leaves room for a non-wellfounded reality. The new physics requires us to believe in a plurality of objects and to accept analyses that appear very unintuitive; these analyses are metaphysical, at least in that they make demands concerning the size and arrangement of the components of reality. Stebbing's naturalism is thus interestingly different from Quinean or Carnapian naturalism, being more metaphysical. Her system brings with it a robust metaphysical apparatus. Though one devoid of necessity, fundamentality, and determination, it includes facts, particulars, and universals (including relations). Stebbing's system and metaphysical views

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remain defensible in the twenty-first century and deserve to be better known to us now.*

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Infinite Regresses, Ground Conditions & Metaphysical Satisfaction

Donnchadh O'Conaill & Olley Pearson

In this paper we clarify a regress argument for metaphysical foundationalism, distinguishing strong and modest versions of this argument. We suggest that while the strong version is open to objection, the modest version is much more plausible and it supports a methodological stance: one ought to refrain from assuming that anti-foundationalism is metaphysically possible. This modest stance follows from our argument that currently we lack reasons to believe anti-foundationalism is possible. This stance opens a new topic in the debate between foundationalism and anti-foundationalism, placing a burden on the anti-foundationalist to provide reasons to think that anti-foundationalism is possible.

A well-known version of metaphysical foundationalism holds that the universe must have a fundamental level, a collection of entities on which all other entities depend and none of which is itself dependent upon anything else for its existence. To deny this, it has been argued, is to set up a vicious regress of dependent entities (e.g., Leibniz 1989, 149–150, 217–218; Fine 1995, 286; Schaffer 2010, 37). However, this vicious regress argument has come under increasing criticism. Whether or not the argument succeeds depends on whether or not the regress in question is vicious, and, notoriously, proponents of vicious regress arguments often simply assert this to be the case (for discussion see Maurin 2007; Bliss 2013).

In this paper, we shall consider one kind of regress. While we do not think it establishes foundationalism, we shall argue that it shows that anti-foundationalism fails a plausible principle of metaphysics, the Principle of Satisfaction: a fact which cannot obtain unless its ground conditions are satisfied should not be assumed to be possible unless there is reason to believe that those conditions can be satisfied.[1] Those who find such a principle plausible

1 The notion of ground conditions shall be defined in the next section.

should be moved by the regress argument to what we term *modest foundationalism*: one ought to refrain from assuming that anti-foundationalism is metaphysically possible. (To be more precise, we shall largely discuss scenarios, each of which contains only a single maximal[2] grounding chain. Therefore, in what follows, by modest foundationalism and anti-foundationalism, we specifically mean modest foundationalism and anti-foundationalism with regard to scenarios of this form, unless otherwise specified.)

In section 1, we introduce the debate and some terminology. In section 2, we present a version of the regress argument that relies on a strong generalising claim from each member of the regress to all of the members of the regress. In section 3, we present a more modest version of this generalising claim and use it to develop a modest version of the argument, supporting an epistemic claim. This epistemic claim in turn supports the methodological claims that anti-foundationalism fails the Principle of Satisfaction and so modest foundationalism ought to be adopted. In section 4, we briefly discuss more complex grounding structures, ones containing multiple maximal grounding chains.

1 Definitions

The dispute between foundationalists and anti-foundationalists has been discussed in terms of a number of different relations (see Tahko 2023). We shall consider this dispute in terms of grounding, following much of the recent literature (e.g., Schaffer 2010; Bliss 2013; Morganti 2014; Dixon 2016; Rabin and Rabern 2016; Trogdon 2018). We shall adopt an orthodox view on which grounding is an irreflexive, asymmetrical, and transitive relation that holds between facts: for the fact that f to be grounded by the fact that g is for f to obtain in virtue of the obtaining of g (hereafter $g < f$).[3] We shall work with a notion of grounding as *partial*, in the sense that it can include both full grounding and merely partial grounding. A full ground for f is by itself sufficient for f to be grounded (Fine 2012, 3); a merely partial ground for f is not.

We understand foundationalism as follows:

2 This will be defined in the following section.

3 Each of these formal features has been questioned, e.g., in Rodriguez-Pereyra (2015); for a defence, see Raven (2013). We shall formulate grounding claims using relational predicates, e.g., f is grounded by g; for discussion of predicational and operational formulations of grounding claims, see Correia and Schnieder (2012, 10–12).

FOUNDATIONALISM. Every non-fundamental fact f is fully grounded by some fundamental fact g or facts, Gs (Dixon 2016, 446; see also Rabin and Rabern 2016, 366).

f is *fundamental* $=_{df}$ there are no Gs such that f is partially grounded by any G (Dixon 2016, 442).

We shall also use the notions of grounding chains and maximal grounding chains, which we define as follows:

A *grounding chain* $=_{df}$ a group of facts, each member of which either grounds or is grounded by each of the other members.

A *maximal grounding chain* $=_{df}$ a grounding chain such that it is not the case that there is a fact that grounds each member of the chain.[4]

It is common to think that some entities are subject to necessary conditions. For instance, plausibly non-empty sets exist only if their members exist; an entity is red only if it is coloured; and an entity is a cube only if it has six sides. In some such cases, the condition is *satisfied* (for instance, each red postbox is coloured); in others, the condition might not be satisfied (for instance, the non-empty set of unicorns does not exist). We believe that all grounded facts are subject to a specific kind of necessary condition concerning their grounds: a grounded fact can obtain only if there obtains some fact or facts that ground it.[5] To capture this idea, we shall now introduce two concepts that will be crucial to our argument in this paper:

C is a *ground condition for* f $=_{df}$ C is a necessary condition for the obtaining of f, which can only be satisfied by the obtaining of facts that ground f.[6]

4 This definition follows Dixon (2016, 453), and Rabin and Rabern (2016, 364).

5 We think that this is a plausible assumption (though in contrast with Wildman 2018). If this assumption turns out not to be true of all grounded facts, our argument can be read as applying to just those grounded facts for which it is true. This highlights a hitherto undiscussed potential anti-foundationalist response to the regress argument, namely, to try to argue that, although the facts involved in the regress are each grounded, some of them do not require grounds in order to obtain.

6 f may have necessary conditions other than its ground conditions. In what follows, we set these other necessary conditions aside. We use the term 'fact' non-factively. Thus, we characterize

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> D is a *total ground condition for* $f =_{df}$ (a) D is a ground condition for the obtaining of f, and (b) there is no condition E that is neither identical to nor a proper part of D and which is a ground condition for f.

We shall not provide a metaphysics of necessary conditions or, hence, of the part-of relation between such conditions. However, instances of these relations holding between grounding conditions can be identified via the satisfiers of those conditions, namely, the grounds of certain facts.[7] A ground condition for the obtaining of f, C*, is a proper part of a ground condition for the obtaining of f, C, iff all the grounds of f that satisfy C* are among the grounds of f that satisfy C, and not vice versa. (The ground condition for the obtaining of f, C, is identical with a ground condition for the obtaining of f, C*, if and only if the grounds of f that satisfy C are all and only the grounds of f that satisfy C*.)

To illustrate these definitions, consider first *h*, the fact that A and B exist. *h* is grounded by *g*, the fact that A exists, and *h* cannot obtain unless *g* obtains. We can describe this case by saying that the obtaining of *g* is a necessary condition, or more specifically, a *ground condition*, C, for the obtaining of *h*, and conversely, that the obtaining of *g* *satisfies* the ground condition C. Now consider *i*, the fact that A or B exists. *i* cannot obtain unless either *g*, the fact that A exists, obtains and grounds *i*, or *j*, the fact that B exists, obtains and grounds *i*. We can capture this by saying that *i* has a ground condition C** that can be satisfied by either the obtaining of *g* or of *j*. Note that unlike C*, the satisfaction of C** does not require the obtaining of any specific fact; it just requires the obtaining of either *g* or *j*. Another difference between these two examples is that C** could be a total ground condition for *i*, but C* could not be a total ground condition for *h*, as *h* also requires the obtaining of *j*, the fact that B exists.

A fact that does not have a ground condition does not stand in need of being grounded; it can obtain without there being any facts that ground it. We assume that all such facts are fundamental facts.

ground conditions as concerning *the obtaining of* facts (though for ease of presentation, we will occasionally drop this phrase and speak simply of *conditions for facts* and of *facts satisfying* those conditions).

7 The obtaining of *g* or the Gs satisfies the ground condition, C, for the obtaining of f, if C is the condition that necessarily f can obtain only if *g* or the Gs obtain and *g* or the Gs ground f, and if *g* or the Gs do obtain and do ground f.

If D is f's total ground condition, then f stands in need of ground (since it has ground conditions), and if D is satisfied, then f's need for ground is completely met; f can obtain without any further grounds of it obtaining.

To further clarify the notion of a total ground condition, it is useful to contrast it with a more familiar notion that we have already mentioned, that of a full ground. To draw this contrast, suppose f is the fact 'Some human exists'. We assume that f is fully grounded in each of its instances. f thus has multiple full grounds, but it has only a single total ground condition. It might be thought that f's total ground condition could be satisfied by any one of f's full grounds, but whether or not this is so depends on further considerations. Consider fact g, the fact that Greta Thunberg exists, and suppose that g is not itself grounded. In that case, f would be fully grounded in g, and g would satisfy f's total ground condition. But now suppose that g is itself grounded, e.g., in certain biological facts. In that case, g could not by itself satisfy f's total ground condition. This suggests a second contrast between a full ground and a total ground condition. While a full ground for a fact f is sufficient for f to be grounded, that full ground might itself be unable to obtain unless itself grounded by further facts, in which case f itself could not obtain unless these further grounds obtain.[8] In contrast, if f's total ground condition is satisfied, no other facts need obtain in order for f to obtain.

The reason for introducing the notions of ground conditions and total ground conditions is that they allow us to focus on what is really at stake in the regress argument—not which grounds a certain fact is posited as having, but which grounds a fact needs in order to obtain. Consider the following toy example: $h < g < f$. In this example, f is posited as having grounds, and these grounds (g and h) together satisfy a ground condition for f, which we can term C*. The question is whether any other facts are needed in order for f to obtain—or, put another way, whether or not C* is a *total* ground condition for f. Whether it is will depend on further information about this example. For instance, assume that h itself has a ground condition. If this ground condition was not satisfied, then h would fail to obtain, in which case C* would not be satisfied and f could not obtain. Therefore, the ground condition for h is also

8 Note also that if a scenario is stipulated as containing a full ground for f, it does not follow that this scenario is possible, since it does not follow that f's total ground condition is satisfied in this scenario.

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a ground condition for f, and C* could not be a total ground condition for f since f has a ground condition in addition to C*.[9]

2 The Strong Argument

In this section, we shall describe a vicious regress argument against anti-foundationalism, which we shall term the *strong argument*. More specifically, it is an argument against anti-foundationalism regarding any scenario in which each fact belongs to just one maximal grounding chain. Having outlined the strong argument, we shall state why we do not accept it, and in the following section, we shall put forward a different, more modest argument.

Both the strong argument and the more modest argument make use of a certain procedure, which we introduce as follows: Consider a scenario in which a fact, f, obtains. If f has a ground condition, then, in order for f to obtain, some other fact or facts must obtain and ground it. Suppose that f is grounded in g, a fundamental fact, and in no other fact. g satisfies f's total ground condition; therefore, no other facts need obtain in order to ground f. Now suppose f is grounded in a non-fundamental fact, g_1. f's total ground condition is not satisfied by g_1, since g_1 itself stands in need of ground. If g_1 is grounded in a fundamental fact, g_2, then g_2 can satisfy g_1's total ground condition, and g_1 and g_2 can together satisfy f's total ground condition.

In what follows, we shall speak of a fact's total ground condition being satisfied *at a point on a chain*, where to say that f's total ground condition is satisfied at a point on a chain, g_n, is to say that f stands in a grounding chain with g_n such that g_n and the facts in the chain which it grounds and which ground f together satisfy f's total ground condition. In the above scenario with f, g_1, and g_2, f's total ground condition is satisfied at g_2.

What we have said so far suggests a procedure that can be applied to any fact f that stands in need of ground and belongs to a single maximal grounding chain: we can go down the chain looking for a point at which f's total ground condition is satisfied. If the chain contains a fundamental fact, g_n, then f's total ground condition will be satisfied at that point on the chain. However, if none of the facts that ground f is a fundamental fact, then f's ground

9 If we further assume that g and h are each full grounds of f, this example makes clear how a single full ground of f may not be itself sufficient to satisfy f's total ground condition. For instance, g would be sufficient to ground f insofar as, if g obtains, nothing else is needed to ground f. But in the scenario just described, g cannot by itself satisfy f's total ground condition because g, and hence f, can only obtain if h does.

condition cannot be satisfied at any point on this chain. This is because any point on the chain will be such that the fact located at that point has ground conditions not satisfied at that point, and those ground conditions will also be ground conditions for f, which are hence also not satisfied at that point. Thus, in such a chain, there is no fact, g_n, such that g_n and the facts that it grounds can together satisfy f's total ground condition. This point holds even if f stands in an infinitely descending maximal chain of grounding.

The next step in the strong argument is the crucial one, and also potentially the most problematic. This is a *generalising claim*, from the preceding claim about each of the facts that ground f to a general claim about them all together. The strong argument makes use of the following:

> The Strong Generalising Claim (Strong Claim). For any grounding chain, if a fact's total ground condition cannot be satisfied at any point in that chain, then it cannot be satisfied by the facts in that chain at all.

Given the Strong Claim, it is not possible for f's total ground condition to be satisfied by the facts in an infinitely descending grounding chain. The same is true for any member of such a grounding chain: there is nothing unique about f in this example. It follows that no such chain is possible. Therefore, any scenario in which each fact is a member of just a single maximal grounding chain must be such that each maximal chain contains a fundamental fact that grounds each member of the chain.

Something like the Strong Claim is found in other vicious regress arguments for foundationalism. For instance, Anna-Sofia Maurin argues that a regress is vicious if the direction of the regress follows what she terms the direction of dependence:

> The regress is vicious because the trigger, to exist (or, the triggering statement, to be true) requires, first, that step one exists (or, is true), which, in turn requires that step two exists (or, is true), etc. *ad infinitum*. The existence of the trigger will therefore depend on the existence of some "final" step of the regress—a step that will never exist given that the regress is infinite. (Maurin 2007, 21)[10]

10 The trigger is whatever starts the regress, e.g., the obtaining of f in the procedure outlined earlier in this section.

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In our terms, we can reconstruct Maurin's argument as follows: If a certain fact (the trigger) has a ground condition, it can only obtain if its immediate ground obtains; its immediate ground can only obtain if *its* immediate ground obtains; and so on. And (this is the STRONG CLAIM) the trigger can only obtain if there is a final step in the regress, a fact that has no ground condition. Both the strong argument and Maurin's argument seem to involve a conditional assumption of the following form: if a certain condition cannot be satisfied at any point in the chain, it cannot be satisfied by the facts in the chain at all.

We shall not rely on the STRONG CLAIM in what follows. In effect, it amounts to the following: the facts that together satisfy f's total ground condition must be located at some point in the chain. That is, if the chain of facts is possible, then at some point in the chain there should be a fact, g_γ, which is such that g_γ and the other members of the chain between it and f together satisfy f's total ground condition. But to assume this is to beg the question against the anti-foundationalist. This is because the kind of chain the anti-foundationalist describes—a grounding chain containing an unbounded infinity of members—is structured in such a way that no member of it could possibly satisfy the description of g_γ we have just given.[11] It may be, of course, that the STRONG CLAIM turns out to be correct. But dialectically, it carries little force against the anti-foundationalist. For the strong argument to work, the STRONG CLAIM must be supported by an independent argument.[12]

3 The Modest Claim and the Principle of Satisfaction

In this section, we turn to a different version of the regress argument, which we term the *modest argument*. It utilises the following claim:

11 A similar point is made in Bliss (2013, 407–408).

12 It is important to note that the demand for the satisfaction of a total ground condition is not a demand that a chain has a fact like g_γ or a termination point. A termination point is a member of the chain that grounds all other members of the chain and that is itself ungrounded, i.e., a fundamental fact. It is clear that a chain containing a termination point can satisfy a fact's total ground condition. But the definition of a total ground condition leaves open the possibility that such a condition could be satisfied by an unbounded infinite chain of grounds: in that scenario, each member of the chain after a given fact, f, would satisfy a ground condition of f, and f would have no other ground condition that needed to be satisfied. (It may turn out to be the case that a chain cannot contain a total ground condition unless it terminates—but this is a substantial further claim, one that, in effect, the strong argument is an attempt to justify. We do not assume that the facts in a chain cannot satisfy a fact's total ground condition unless the chain terminates, and we contend that the strong argument does not succeed in establishing such a claim.)

> The Modest Generalising Claim (Modest Claim). For any grounding chain, if a fact's total ground condition cannot be satisfied at any point in that chain, then we lack reason to believe that it can be satisfied by the facts in that chain at all.

The Modest Claim is very plausible. As we argued in section 2, in a chain with no fundamental fact, f's total ground condition cannot be satisfied at any point in the chain. We accepted at the end of the previous section that it does not straightforwardly follow from this that f's total ground condition cannot be satisfied in this scenario. But we have no reason to believe that it *can* be satisfied, because it is not clear what else there is in this scenario to which the anti-foundationalist can appeal in order to satisfy f's total ground condition.[13] (Note, our claim is not that it is impossible to provide such reasons, but that after a careful consideration of a putative anti-foundationalist ontology, as yet none are forthcoming.)

We anticipate two responses to the Modest Claim. The first is that it overlooks the possibility of appealing, not to *any* specific point on the chain, but to *all* of the facts in the chain together (or more specifically, to all of the facts in the chain, each of which grounds f). In other words, the suggestion is, the Modest Claim commits something like a fallacy of composition: moving from a true claim about each member of the chain to a false claim about all members of the chain.

This response would work against the Strong Claim (indeed, it is very similar to the criticism of the Strong Claim we offered at the end of the previous section). But it is not convincing as a response to the Modest Claim, precisely because the latter is a weaker claim. The Modest Claim, to repeat, is that we lack reason to think that the facts in the chain can together satisfy f's total grounding condition. In other words, to affirm the Modest Claim is not to rule out that all of the facts in the chain are together able to satisfy f's total grounding condition; it is to claim that we have no reason to think that all of the facts in the chain are capable of doing so. It is true that claims about all of the facts in the chain are logically distinct from claims about any of the facts in the chain, but this truth does not by itself provide reason to think that all of the facts in the chain can together satisfy f's total grounding condition. To undermine the Modest Claim, the anti-foundationalist requires something more than this logical difference; she owes us a reason to think that there

13 The same will be true of any member of such a chain, as there is nothing unique about f here.

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is an *ontological* difference, i.e., that all the facts in the chain together can satisfy f's total ground condition. (More precisely, she owes us an argument that this is possible as opposed to simply stipulating that it is, since such a stipulation would beg the question in favour of anti-foundationalism.)

A second response to the MODEST CLAIM might appeal to the point that every fact in an infinitely descending chain has a full ground; in such a chain, every fact needing a ground has one, so all total ground conditions must be satisfied. But this response is inadequate. As was mentioned in footnotes 8 and 9, that a postulated scenario contains a full ground for a fact f does not entail that it contains facts adequate to satisfy f's total ground condition. Therefore, one cannot directly argue from the claim that every fact in a maximal chain has a full ground to the conclusion that every fact in this chain has its total ground condition satisfied. For it to be clear that a scenario is one in which the total ground condition for f was satisfied, it would have to be clear that in this scenario none of f's ground conditions was not satisfied. But this does not follow from the fact that in the scenario some of f's ground conditions are satisfied (which is all that straightforwardly follows from each fact having a full ground).

The *modest argument*, as we shall refer to it, combines the MODEST CLAIM with the claim defended in the previous section that f's total ground condition cannot be satisfied at any point in the chain. Together, these claims support an epistemological conclusion: we lack reason to believe that f's total ground condition can be satisfied by the facts in the chain. As noted above, it would be a mistake to infer from this that anti-foundationalism is false. But one can infer a more modest methodological conclusion:

> MODEST FOUNDATIONALISM. One ought to refrain from assuming that anti-foundationalism is metaphysically possible.[14]

MODEST FOUNDATIONALISM follows from the modest argument via the following methodological principle:

> THE PRINCIPLE OF SATISFACTION (PS). A fact that cannot obtain unless its grounding conditions are satisfied should not be assumed

14 Again, we are limiting ourselves for the time being to scenarios with only a single maximal grounding chain. We discuss scenarios containing multiple maximal chains in section 4.

> to be possible unless there is reason to believe that those conditions can be satisfied.[15]

We shall not be able to provide a thorough defence of PS, but we shall outline a general motivation for accepting it. Whilst it can be appropriate to assume that certain facts are possible, the obtaining of a grounded fact is conditional on certain necessary conditions, specifically its ground conditions, being satisfied. PS spells out an approach that one ought to take towards the possibility of such facts in light of their having ground conditions.

To see how this works, consider an example that does not obviously involve issues to do with non-well-foundedness: the possibility that the singleton set {Pegasus} exists.[16] Call the fact that {Pegasus} exists f.[17] It seems plausible that if f would obtain, it would be grounded in the fact that Pegasus exists, and furthermore, unless Pegasus existed, f's total grounding condition could not be satisfied. Applying PS, we suggest that one should not accept that f is possible unless one has reason to believe that it is possible that Pegasus exists. This seems like a perfectly reasonable approach to take. Conversely, it seems unreasonable to accept that f is possible if one has no reason to believe that it is possible that Pegasus exists.

Consider another example: the possibility that some humans are immortal. Call the fact that some humans are immortal g. One might think that if g obtains, it would be grounded by one or more of its instances, i.e., by the fact that a specific human, Nigel, is immortal (call this fact h). But in order for this to provide a reason to think that g is possible, we surely need some reason to think that h is possible. If we have no such reason, then it is surely unreasonable to justify the thought that g is possible by postulating g's being grounded in h. Alternatively, one might think that if g obtains, it would be grounded in, e.g., certain biological facts; but again, without any reason to think that these biological facts are themselves possible, it seems unreasonable

15 As Bliss (2013, 415) notes, it might be possible to motivate a regress argument against anti-foundationalism using a Principle of Sufficient Reason (PSR). It is worth noting that PS is much more modest than a PSR. Whilst a PSR demands that everything in a scenario requires an explanation, PS only says that, regarding entities that we have reason to believe are impossible unless certain conditions are met, we ought not to postulate these entities in a scenario unless we have reason to believe that those conditions can be met in that scenario.

16 Thanks to two referees for suggesting this example and, more generally, for suggesting that we need to spend more time motivating PS.

17 Recall that we are using a non-factive conception of facts, so we are not committing ourselves to f's actually obtaining or even to its being metaphysically possible.

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to think that g is possible. PS is in effect a generalisation of these specific claims: if you think that a fact has grounding conditions, and if you lack reason to think that any of the facts that would ground it are possible, then you should refrain from accepting that this fact is possible.[18]

PS is a claim concerning modal epistemology, specifically regarding whether or not we should accept that certain scenarios are metaphysically possible. A thorough analysis of PS would require discussing how it relates to various existing approaches in modal epistemology.[19] We shall not be able to address this topic in the detail it deserves, but we shall consider how PS relates to one well-known approach: appeals to conceivability. Again, it will help to start with a toy example that does not involve non-well-foundedness: whether or not it is possible for pigs to fly. One way to address this is to ask whether or not this scenario is conceivable, and a simple claim is that if it is conceivable (or conceivable in a certain way), then we have reason to think that it is metaphysically possible.[20] On the face of it, this approach does not require applying PS and indeed even seems to rule it out (whether a grounded fact, f, is possible would simply be settled by whether we could conceive f itself).

However, even if it is true that PS does not align straightforwardly with the conceivability approach, it is not necessarily at odds with it. To make this clear, assume that the fact that there is a flying pig (call this fact f) has a ground condition (if it does not, then PS would not apply to it). Given this, for the scenario conceived of to be metaphysically possible, it must contain facts that ground f and that together satisfy f's total ground condition (this is something that even the conceivability theorist should be willing to accept).

PS entails that we should not regard this scenario as possible unless we have some reason to believe that it contains facts that satisfy f's total ground condition. Whether or not this restriction is compatible with the conceivability approach will depend on how demanding a notion of conceiving is appealed to. Suppose that conceiving of a scenario only justifies one in thinking that it

18 Note that PS leaves open what can count as a reason to accept that a fact's grounding conditions can be satisfied. This is not a problem. PS is not intended to be a method for discovering which scenarios are (or are not) metaphysically possible. Rather, it is intended as a constraint to be applied to claims that certain scenarios are possible. As a comparison, consider appeals to testimony. As a general rule, one should not accept testimony as a reason to believe p unless the testimony is from a reliable witness. This seems to us to be a perfectly good epistemic rule, but it leaves open exactly what standards must be met in order for someone to be a reliable witness.

19 Thanks again to two referees for suggesting that we engage with this literature.

20 There are a number of different ways of conceiving a scenario (Chalmers 2002). In what follows, the differences between them will, for the most part, not be relevant.

is possible if one conceives of it in exhaustive detail. PS is perfectly compatible with this kind of appeal to conceivability: a clear conception of how the facts in a scenario satisfy *f*'s total ground condition would qualify as a reason to think those facts can satisfy its total ground condition. Suppose, on the other hand, that conceiving a scenario is supposed to justify one in thinking it is possible, even if one's conception omits or glosses over many important details of the scenario. This kind of appeal to conceivability may not be compatible with PS. However, we suggest there is an independent reason to be sceptical of this kind of appeal to conceivability. One well-known advocate of such scepticism is Peter van Inwagen, who notes that "to assert the possibility of *p* is to commit oneself to the possibility of a whole, coherent reality of which the truth of *p* is an integral part" and suggests that conceivability theorists often do not examine the details of such proposed realities (1998, 78). It has also been argued that conceivability is not a reliable guide to possibility insofar as it involves simply stipulating certain features of the conceived scenario (e.g., Kung 2010; Berto and Schoonen 2018). While we shall not defend these more sceptical approaches to conceivability in any detail, they illustrate that there is existing work on modal epistemology that is at least compatible with PS.[21]

An anti-foundationalist may respond that all the modest argument shows, even in conjunction with PS, is that if one is going to assume that a possible world contains a grounded fact, one must also assume that it contains everything necessary to satisfy that fact's total ground condition. To this end, she might add an assumption to her position: a maximal grounding chain contains all of the facts needed to satisfy *f*'s total ground condition.

However, this response is inadequate. What is precisely at issue is whether or not the total ground condition for a fact is satisfied by the facts in a specific kind of chain, e.g., an infinitely descending maximal grounding chain. We submit that the anti-foundationalist is not entitled to assume that they are without further argument. We have already provided reasons to think that *f*'s total ground condition cannot be satisfied at any point in an infinite grounding chain, and as argued above, it is not clear what else in the chain could satisfy *f*'s total ground condition. Therefore, the anti-foundationalist needs to provide some reason to think that the facts in such a chain would contain facts capable

21 We accept that not everyone will be satisfied with our discussion of how PS relates to existing work in modal epistemology; in particular, someone who thinks there are independent reasons to accept appeals to conceivability may be tempted to reject PS on this basis. Our defence of PS can therefore be understood as conditional: one should accept PS unless one already has reason to accept an approach in modal epistemology that undermines it.

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of satisfying f's total ground condition. Until such further reason is provided, we ought to refrain from assuming that f's total ground condition would be satisfied by any or all facts in such a chain. And this leads immediately to refraining from assuming that such a chain is possible.

Combining PS with the modest argument leads us to MODEST FOUNDATIONALISM. MODEST FOUNDATIONALISM does not entail the falsity of anti-foundationalism. Rather, it is a methodological stance towards anti-foundationalism; one should not assume that anti-foundationalism is either actually or possibly true. This stance is open to revision, but the burden lies with the anti-foundationalist to provide some positive reason to think that what she is describing is metaphysically possible (we have argued that currently we are lacking any such reason).[22]

4 Other Grounding Structures

The argument we have given so far establishes MODEST FOUNDATIONALISM for scenarios in which each fact belongs to a single maximal grounding chain. We believe that essentially the same argument can be given for scenarios where facts belong to multiple maximal grounding chains. We do not have adequate space here to make this more general argument, but in this section, we will say something to indicate what form it would take.

The argument we have given for facts belonging to single maximal chains works by establishing that at no point in such a chain will we locate grounds adequate to satisfy a fact's total ground condition: call this claim SINGLE. The argument then moves from SINGLE via the MODEST GENERALISING CLAIM and PS. To make the more general argument, we need an analogue of SINGLE for complex structures where facts belong to more than one maximal grounding chain. We propose the following: at no level in such a structure will we locate grounds adequate to satisfy a fact's total ground condition (where a level simply consists of one point on each of the maximal chains to which the fact belongs).[23] We will refer to this claim as COMPLEX. We would also

22 Cameron (2008, 12–13) also argues for a position more modest than foundationalism as usually understood, and he also utilises a methodological principle; but his position and the principle he uses each differ from ours. Cameron's argument is essentially that we ought to take the actual world to be foundationalist because it permits unified explanations. Our argument is that we ought not to assume that any possible world is anti-foundationalist, because we lack reason to believe that anti-foundationalism can satisfy any fact's total ground condition.

23 The notion of a level allows for the concern that we might need to consider points on more than one maximal chain to locate facts adequate to satisfy a fact's total ground condition. Our talk of a

need an analogue of the MODEST CLAIM, and we propose the following: for any grounding structure, if a fact's total ground condition cannot be satisfied at any level in that structure, then we lack reason to believe that it can be satisfied by the facts in that structure at all. We will refer to this as the SECOND MODEST CLAIM.

In order to establish MODEST FOUNDATIONALISM for complex structures, we would try to establish COMPLEX and move from it via the SECOND MODEST CLAIM and PS. We would hope to justify the SECOND MODEST CLAIM much as we have justified the MODEST CLAIM: it is not clear what else could satisfy the total ground condition of a fact in such a structure other than the levels in the structure.

Whilst we can't argue for COMPLEX here, we believe that it is actually a claim that many anti-foundationalists would be willing to accept. This is because it seems the most apt anti-foundationalist scenarios would include continuous grounding series, and such continuous series are specifically ones where we do not locate total ground conditions at any level.

Before closing this section, we will briefly consider three complex grounding structures in order to show how our arguments apply to them. We do not intend this to establish the general argument but to indicate how it can handle specific kinds of complex grounding structure.

In the first scenario, f is fully grounded in a fundamental fact, g. f is also fully grounded in h_1, which is fully grounded in g and also fully grounded in h_2. h_2 is fully grounded in g and also in h_3, etc. That is, the Hs form an infinite descending chain of grounding such that each of the Hs, h_δ, is itself fully grounded in g as well as in the subsequent H, $h_{\delta+1}$. The facts in this scenario together form a *fully pedestalled chain* (Dixon 2016, 447–448).[24] This kind of grounding structure should be acceptable to a foundationalist because each non-fundamental fact in it is fully grounded in a fundamental fact, g.

The generalised argument concerning complex structures that we outlined above accommodates this scenario. Starting with f, our argument requires that we must be able to locate a fact or facts adequate to satisfy its total ground condition at some level in the structure. We can easily do this. In particular, g can satisfy f's total ground condition. To see this, we can note

level should not be taken to imply that facts adequate to satisfy a fact's total ground condition must be satisfied by some point on *each* maximal chain.

24 All of the facts in this scenario belong to a single maximal chain. However, f also lies on multiple maximal chains in the structure (e.g., $g < f$, $g < h_1 < f$, etc.), so it counts as complex and relevantly different to the scenarios discussed previously.

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that g is sufficient for f to be grounded and g does not itself have any ground conditions, so the ground condition for f that is satisfied by g is a total ground condition for f.[25] Therefore, any level in the structure that includes g will be a level at which we can locate facts adequate to satisfy f's total ground condition. Furthermore, the same holds for each of the other non-fundamental facts in the scenario, i.e., each of the Hs. For example, g can satisfy h_1's total ground condition in a way analogous to that in which it satisfies f's. Thus, COMPLEX does not arise in this scenario, and so our argument does not stand against it.[26]

In the second scenario, f is the fact that A exists or B exists. f belongs to two maximal grounding chains. The first is $g < f$, where g is the fact that A exists. The second is $\ldots < i_3 < i_2 < i_1 < f$, where i_1 is the fact that B exists, i_2 is a fact that fully grounds i_1, i_3 fully grounds i_2, etc. g is fundamental and stands in no grounding relation with any of the Is.

Focusing on f, our generalised argument requires that we must be able to locate facts adequate to satisfy its total ground condition at some level in the structure. And it would seem that we can do this. As above, it seems g can satisfy f's total ground condition.

However, while we can locate grounds adequate to satisfy f's total ground condition, this is not true of any of the other non-fundamental facts in the structure. For example, i_1 lies on a single maximal chain of grounding, which contains no fundamental fact. Thus, a variant of COMPLEX arises in this second scenario, and our argument applies against it, as the foundationalist would want.

In the third scenario, f is merely partially grounded in a fundamental fact, g. f is also merely partially grounded in h_1, and together g and h_1 fully ground f. h_1 is merely partially grounded in g and merely partially grounded in h_2, and together g and h_2 fully ground h_1. h_2 is in turn merely partially grounded

25 We do not intend our comments in the sentence to generalise; that is, we are not implying that in any scenario whatsoever, any fact that is fundamental and a full ground of f will satisfy f's total ground condition. For example, consider an adjusted first scenario, which is as the first scenario except g is fully grounded in g_1, which is fundamental. In the adjusted first scenario, g_1 would not satisfy f's total ground condition. However, g_1 would satisfy g's total ground condition, and hence g_1 and g together would satisfy f's total ground condition. Likewise for any variation of this scenario in which f stands in a maximal chain of full grounding, which includes a fundamental fact.

26 The first scenario illustrates the claim made in footnote 12 above that the demand that each non-fundamental fact has its total ground condition satisfied is distinct from the demand that all chains of grounding must terminate.

in g and merely partially grounded in h_3, and together g and h_3 fully ground h_2. And so on, so the Hs form an infinite descending chain of grounding such that each of the Hs, h_δ, is itself merely partially grounded in g as well as in the subsequent H, $h_{\delta+1}$, such that g and $h_{\delta+1}$ together fully ground h_δ.

The facts in the third scenario together form a *partially pedestalled chain* (Dixon 2016, 454–455; see also Pearson 2023). Our arguments apply against this kind of grounding structure, as a foundationalist would want. Because g is a merely partial ground of f in this structure, g cannot by itself satisfy f's total ground condition. Further, each of the other facts in the structure itself requires grounds, and so we cannot locate facts adequate to satisfy a total ground condition for f at any one of these either. For example, h_2 satisfies a ground condition for h_1, and so h_1 alone cannot satisfy a total ground condition for f, and h_3 satisfies a ground condition for h_2, so h_1 and h_2 together cannot satisfy a total ground condition for f, and so on.

The points just noted also undermine our ability to locate facts adequate to satisfy f's total ground condition at any level in the structure. For example, if we consider the level made up of g and h_1, together these are sufficient for f to be grounded, but they cannot satisfy f's total ground condition since h_1 itself has further ground conditions: h_1 cannot obtain unless h_2 does, and so f cannot obtain unless h_2 does. And so on for the level made up of g and h_2 together, and the one made up of g and h_3 together, etc. Thus, COMPLEX arises in the third scenario, and so our argument stands against it.

5 Conclusion

We have suggested that the vicious regress argument for foundationalism can be understood in two ways: as containing a strong or a modest claim. The STRONG CLAIM will likely be something the anti-foundationalist denies. The MODEST CLAIM, together with PS, supports MODEST FOUNDATIONALISM for facts lying on single maximal grounding chains. MODEST FOUNDATIONALISM can also be shown to hold for at least some complex scenarios where facts belong to multiple maximal grounding chains. This position opens a new topic in the debate between foundationalism and anti-foundationalism; it places a burden on the anti-foundationalist to provide reasons to think that anti-foundationalism is metaphysically possible.*

* We have been working on this paper, or ancestors of it, for a very long time, so we apologise in advance if we omit any names that should be included here. Thanks to Sarah Adams, Einar

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A Recipe for Non-wellfounded but Complete Chains of Explanations (And Other Determination Relations)

Alexandre Billon

Consider a chain of explanation $u_1, u_2, u_3, \ldots$ where each item is fully explained by its successor. Suppose that the chain is non-wellfounded in the sense that it is either infinite or circular. Explanations here can be either causal explanations or grounding, metaphysical explanations. It is widely believed that such chains of explanation will always be incomplete, i.e. leave things to be explained, and many philosophers have considered this as a vice of non-wellfounded explanations, a vice they invoked to rule out such explanations. In this article, I argue that in some cases non-wellfounded are in fact complete. I also put forward both necessary and non-trivial sufficient conditions for the completeness of non-wellfounded explanations.

Consider a series $(u_i)_{i\in I}$, whose items are each (fully) explained by their immediate successor. I can be: (a) the set of the first n non-null integers $[\![1, n]\!]$, in which case $(u_i)_{i\in I}$ constitutes a finite, non-circular chain of explanations; or (b) the set of non-null natural numbers $\mathbb{N}^*$, in which case $(u_i)_{i\in I}$ constitutes an *infinite chain of explanations*. (c) I can also be the ring of integers modulo n, $\mathbb{Z}/n\mathbb{Z}$ (you can picture this as the numbers $1, 2, \ldots, n$ sequentially distributed on a circle, just like the numbers $1, 2, \ldots, 12$ are sequentially distributed on a watch dial),[1] in which case $(u_i)_{i\in I}$ constitutes a *circular chain of explanations*.

We will say that in case (a), but not in cases (b) and (c), the chain is well-founded. Let us say, moreover, that a chain of explanation is *complete* when it leaves nothing to be explained (more on this below).

1 The ring of integers modulo n ($\mathbb{Z}/n\mathbb{Z}$) is the set of the n first integers, $1, \ldots, n$ endowed with the addition and multiplication operations, and where (to put it rather roughly) it is assumed that for all x, $x = x + n$.

In a previous article on cosmological arguments, I have put forward a few examples of complete infinite and circular explanations, and I argued that complete, non-wellfounded explanations such as these might explain the present state of the world better than their well-founded theistic counterparts (Billon 2023). Although my aim was broader, the examples I gave there implied merely causal explanations. In this article, I would like to do three things:

- Specify *some general informative conditions for complete and incomplete, non-wellfounded causal explanations* that can be used to assess candidate explanations and generate new examples of complete, non-wellfounded explanations.
- Show that these conditions, which concern chains of causal explanations, easily *generalize to chains of metaphysical, grounding explanations* and even to chains involving other "determination relations," such as supervenience.
- Apply these general conditions to the recent debates against the existence of non-wellfounded chains of grounds and show, with a couple of precise examples, that the latter can be complete and that just like in the case of causal explanations, non-wellfoundedness can, in fact, be an asset rather than a liability.

In the first section, I present the recent debates about non-wellfounded chains of grounds and show more broadly why the question of complete, non-wellfounded chains of explanations is important. I then articulate the framework within which I will assess these questions about non-wellfounded explanations and determination relations (section 2). After that, I reconstruct an argument from Leibniz, which is, I believe, the most interesting argument against complete, non-wellfounded explanations (section 3). This argument rests on a clear example of a non-wellfounded, incomplete explanation. My answer to it rests on clear examples of complete, well-founded explanations (section 4). My examples involve causal (as opposed to metaphysical, grounding) explanations, but in the next sections (sections 5–6), I will put forward general formal conditions a non-wellfounded explanation must meet in order to be complete. These general criteria will then allow me to introduce examples of complete and incomplete, non-wellfounded chains of *grounds* (section 7). In the remainder of the article, I discuss a couple of objections. First, I argue that even though one might quibble about the definition of a *complete* explanation and argue that in the examples put forward our ex-

planations still implicitly leave some things to be explained, these examples unambiguously show that *non-wellfounded explanations can do better than their well-founded counterparts* and that there might be non-wellfounded explanations that leave nothing at all to be explained (section 8). I also show that complete, non-wellfounded explanations are analogous and no less problematic than well-accepted explanations, such as equilibrium explanations and essentialist explanations (section 9). Finally, I discuss the possibility of infinite explanations that are as simple as (or even simpler than) finite, well-founded explanations (section 10).

1 Completeness and Non-wellfoundedness

We all wish we could have complete explanations of some things: explanations, that is, leaving nothing to be explained. Such explanations are the Grail of metaphysical inquiries (think of Leibniz 1989's search for the radical origin of things) but also of scientific inquiries (think of Einstein's quest for a "theory of everything," see Schilpp 1949, 63; see also Hawking and Mlodinow 2010, 181). In their vast majority, however, researchers believe that if such explanations exist, they must be well-founded. This is true in the case of causal explanations, but it has recently come to the fore in the context of debates concerning metaphysical grounding explanations.

An interesting objection, or cluster of objections, against the existence or the very possibility of non-wellfounded chains of grounds centers indeed on the idea that they would be somehow explanatory defective because they cannot be complete. Fine (2010) has, for example, claimed that in cases such as (b) and (c), u_1 would not have a completely satisfactory explanation:

> There is still a plausible demand on ground or explanation that we are unable to evade. For given a truth that stands in need of explanation, one naturally supposes that it should have a completely satisfactory explanation, one that does not involve cycles and terminates in truths that do not stand in need of explanation. (Fine 2010, 105)

Most often, this objection seems to appeal, more or less implicitly, to a version of the principle of sufficient reason (PSR), to the effect that everything must have a (full) explanation. Thus, Schaffer (2010) claims that:

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> There must be a ground of being. If one thing exists only in virtue of another, then there must be something from which the reality of the derivative entities ultimately derives. (Schaffer 2010, 37)

As I understand it, the objection is that non-wellfounded chains of grounds are incomplete in that they leave something to be explained, which is bad by the PSR.

Against this explanatory deficiency objection, advocates of non-wellfounded grounding have argued that well-founded grounding chains face the very same explanatory problem: in case (a), the last item u_n of the series seems in need of an explanation too, and this explanation is lacking (Bliss 2014; Bliss and Priest 2018, 20–21). Yet this "*tu quoque*" reply might be disputed by philosophers who believe that some items are by their very nature somehow self-explanatory, or, at least, "autonomous" in the sense that they do not call for an explanation (see Dasgupta 2016; see also Miller 1996 and other Theists on the existence and simplicity of God).

Various philosophers have recently tried to precisely assess whether, as suggested by Schaffer, Fine, and others, non-wellfounded grounding chains need to fare worse explanatorily than well-founded ones (see the contributions in Bliss and Priest 2018 and the very useful introduction). Some, such as Bliss and Priest, seem to assume that non-wellfounded explanations will never be complete (Bliss 2013, 408; 2019; Priest 2014, 187; Cameron 2022, 130) but reject the request for a complete explanation. Others have underlined the fact that arguments for the incomplete character of non-wellfounded explanations are often unsound or simply lacking (Oberle 2023). Although they bring up interesting points, these discussions remain at a very abstract level and never rely on concrete examples of would-be complete explanations.

2 A Framework for Explanations

I will provide such concrete examples. Before that, let me make a couple of terminological points and set up the framework. I will talk, as I just have, as if grounding *were* metaphysical explanation. This might be disputed. Just like on some views, causality underlies (but differs from) natural explanations, on some views, grounding only underlies metaphysical explanations. Likewise, I will often talk as if explanation were a relation (rather than, say, a sentential operator). I am not particularly keen on the views mirrored by these ways of

talking, but I believe that nothing substantial depends on them here, and they will make my arguments (and my prose) much more fluid.

For simplicity, I will suppose that the relata of the explanation relation (and hence our "items") are facts or sets of facts, where a fact is understood liberally as the referent of a true proposition. To make my prose more fluid and discuss some texts that seem committed to that view, I will sometimes speak as if the relata of explanations could be tropes or individuals. It should be clear, however, that by "trope/individual x explains trope/individual y," I only mean that the identity and/or existence of the former explains that of the latter. Except otherwise noted, by "A explains/grounds B," I will always mean "A *fully* explains/grounds B."

More substantially, I will admit that basic explanations in which one item explains another (as opposed, e.g., to complex chains of simple explanations) have a triadic structure, involving:

- a "final" item,
- an "initial" item,
- and a link accounting for the transition from the final item to the initial item, which I will consider to have the form of a law.

The final item, along with the laws, explains the initial item.[2]

By accounting for the transition between the final and the initial item, the laws do the explanatory work. On some accounts, the link between the items plays no genuine explanatory role or does not have a lawlike structure. I will ignore them here.[3] We shall see, in any case, that my understanding of lawhood is extremely minimal. The triadic framework is less orthodox in the grounding literature than in the causal explanation literature. In the former, it is associated with the works of Schaffer (2016, 2017), Litland (2017), Bader (2017), Kment (2014), Glazier (2016), and Rosen (2017).

In our series $(u_i)_i$, each item u_{i+1} explains, along with the law L_i, the antecedent item u_i. Once laws are introduced, it is natural to wonder whether some laws can themselves be explained by more basic laws (as when we explain the laws of thermodynamics by those of statistical mechanics). Analogously, and this hypothesis shall prove very important in what follows, we

2 So, the final item explains the initial item (and not the other way around). The terminology is a bit awkward here, but it has to be so because the main focus of this paper is infinite *descending* chains of explanations.

3 As emphasized by Schaffer (2017, 308), it is difficult for these accounts to understand the role explanations play in making sense of the world.

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might wonder whether laws can explain some items all by themselves, that is, without the final item—call that cases of zero-explanation or pure-law-explanation.[4] In the literature on the "sublime question"—*Why is there anything? Why this?*—many atheists have, for example, looked for answers that only mention laws (see Nozick 1981, chap. II; Leslie 1979). In a couple of recent articles, Kappes (2022, 2023) distinguishes a restrictive sense of "explain" (in which only the initial item) from a more inclusive sense of explain (in which laws can also be said to explain something). He also argues that the first one corresponds more closely to because-statements. I am not completely sure about this last linguistic claim, but if there really are two senses here to differentiate, then it is definitely the inclusive sense I will be using throughout this article.

The triadic structure of explanation also allows us to make a distinction that we have omitted and that can prove useful in certain contexts. If we want to be very rigorous, we should not identify, as we have until now, *a series of items* such as $(u_i)_{i\in I}$ in which items are explained by their successor, with *a chain of explanations*. A chain of explanations is rather a series of items *and* a series of laws accounting for the transitions between items (or, equivalently from a formal point of view, a series of triplets containing a final item, a law, and an initial item). For convenience, the laws are often left implicit when they are not the target of our explanation or when we are not dealing with zero-explanations. We will follow this convention and often talk as if our series of items $(u_i)_i$ were, by itself, a chain of explanation.

With this triadic characterization of explanation, we can also define *complete* chains of explanations a bit more precisely. As I have used the term, a chain of explanation is complete when it leaves nothing to be explained (concerning the chain of explanation) except the laws. We can call "ultimate" or "supercomplete" a chain of explanations that leaves nothing at all to be explained (concerning the chain of explanation), not even the laws it relies on.

Unfortunately, these definitions are neither very informative nor very useful by themselves. It is probably hard to see, while reading them, why there should be some non-wellfounded series of explanations that are not complete (and Hume is widely held to have claimed that there could be none; see Rowe 1970). We shall see that this is not the case, but it will take a bit of work.

4 Authors who deny that grounding involves laws nevertheless have an analogon of our "metaphysical explanation by the laws alone," namely, what Fine (2012) calls zero-grounding.

3 Leibniz's Argument Against Complete Non-wellfounded Explanations

Leibniz has put forward what I take to be the most interesting argument against the completeness of non-wellfounded explanations. Although Leibniz is not exactly concerned with what we call "causal explanations" or "metaphysical explanations," but rather with "explanations by reasons" (which seems to include causal and teleological explanations), we shall see later that his argument (and my reply to it) have purely causal and ground-theoretic analogs.

3.1 *Against the Completeness of Infinite Explanations*

While exposing his version of the cosmological argument, to the effect that there must be a complete explanation of things and that these necessarily involve God, Leibniz puts forward an objection against the idea that infinite chains of explanations could be complete.

> Let us imagine the book on the *Elements of Geometry* to have been eternal, one copy always being made from another; then it is clear that though we can give a reason for the present book based on the preceding book from which it was copied, we can never arrive at a complete reason, no matter how many books we may assume in the past, for one can always wonder why such books should have existed at all times, why there should be books at all, and why they should be written in this way. (Leibniz 1989, 486)

Why does Leibniz think that this explanation is incomplete? Prima facie, one might think that his three questions can be answered easily by the proposed infinite explanation. "Why [should] such books […] have existed at all times[?]" Well, for each book, we can answer that it exists because of a former copy and because of a scribe who copied it. This, it seems, can provide a satisfying answer. "Why [should] there […] be books at all[?]" Well, because the scribes are instructed to make books out of other books. "Why [should] they […] be written in this way[?]" Because the scribes are instructed to make faithful copies of the book they are given.

The key to understanding Leibniz's objection, I take it, is to distinguish *each book in the series* (Leibniz's "present book" and "preceding book") from *the*

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whole series of books (Leibniz's "such books [...] have existed at all times"). In Leibniz's example, each book copy is explained in terms of its successor in the series (and the law that specifies the behavior of the scribes). But the whole series of books isn't. Indeed, it could be the case that each scribe faithfully copies the next book, as specified by the law, but that the books are all copies of the Bible rather than the *Elements of Geometry*. This suggests that the infinite explanation here does not explain why we have an infinite series of the *Elements of Geometry* rather than an infinite series of the Bible. But if this is so, it clearly leaves something unexplained, then, namely, the whole series itself.[5]

Another reason one might want to add in order to deny that the explanation is complete is that the latter can explain the content of the book copies but not, say, whether they are made of paper or parchment, the color of the cover, or even that they exist—call that the extra-property objection. I'm not sure, however, that this second objection against the completeness of the explanation is really decisive. For it could easily be answered by enriching the laws and specifying that the scribes make a book copy *of the same material, the same color, and with the same book cover* as the book he is given and that *they make it out of an infinite stock of material and ink*, or even out of nothing. These additions would allow one to explain the material, the color of the cover, and the existence of *each* book copy given the next one. It would not, however, allow one to explain why *the whole series* is made of white paper or even exists, but that is another problem. It is, in fact, the very problem we have dealt with in the preceding paragraph.

This second, extra-property objection could equally be answered by specifying the target of the explanation more precisely than Leibniz did and claiming that the items of the series are not full books but merely the content of book copies (understood as facts of the form 'The content of book #*i* is that of the *Elements of Geometry*'), or even, if you are really suspicious about explanations of existence,[6] by claiming that they are conditional facts concerning the

5 We thus have a simple counter-example to the so-called Hume-Edwards principle to the effect that the whole is sufficiently explained in explaining the cause of the parts (Hume 1907). The name of the principle comes from Rowe (1970). See Billon (2023, 1938, especially n. 7); see for a defense of the Hume-Edwards principle against other objections, Şimşek (2023, sec. 3).

6 A referee for this journal suggests that laws being abstract, they cannot explain the existence of concrete objects. I agree that the fact that abstract laws can regiment concrete events can seem puzzling, but I am not sure that this puzzle concerns existence specifically (how can Newton's laws "act on" this rock to make it fall on the ground?). There is, in fact, a long and influential tradition of positing laws or principles to explain existence (see Leslie 1979; 2003—and the

content of book copies (such as 'If book copy #*i* exists, its content is that of the *Elements of Geometry*'). Either by enriching the laws or by impoverishing the items, we can easily dispose of the extra-property objection.

We can now conclude this discussion of Leibniz's infinite scribes case with two important conclusions. First, in order to be complete, a series of explanations $(u_i)_i$ must explain not only each item of the series from its successor but also the whole series itself from the laws alone. Conversely, of course, a chain of explanation that explains the whole series of items will be complete—what is left to explain but the laws once the whole series is explained? So,

> Complete Explanation. A chain of explanation $(u_i)_i$ is complete if it explains not only each item from its successor but also the whole series $(u_i)_i$ from the laws alone.

This is already an interesting characterization of completeness. It shall prove quite useful.

Second, some infinite chains of explanations are not complete. This is the case of the *Infinite Scribes* series, which we can reconstruct as follows:

> *Infinite Scribes.* Consider an infinite series of book copies. Book copy #1 is a copy of the *Elements of Geometry*; its content is explained by the fact that it was copied before by a scribe from an older book copy #2. This older book copy #2 is a copy of the *Elements of Geometry*; its content is explained by the fact that it was copied, before, by a scribe from a yet older book copy #3, and so on to infinity… where a scribe is someone who makes a faithful copy of the book he is given.
>
> Here, u_i is the fact that book copy #*i* (i.e., the book copy that appears at stage #*i*) is the *Elements of Geometry*. And the law L_i specifies the behavior of the scribe #*i*: he makes a new book with the same content as the next book (as all scribes behave in the same way, all the L_is are actually identical).

historical references within—on the axiarchic principle; or Nozick 1981, chap. II on the principle of fecundity), a tradition that still has quite respectable representatives today (Parfit 2011, 623–648 is a notorious example).

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3.2 Against the Completeness of Circular Explanations

Interestingly, this objection to the completeness of infinite explanations generalizes to circular explanations:

> *Circular Scribes.* Consider two book copies. Book copy #1 is a copy of the *Elements of Geometry*; its content is explained by the fact that it was copied in 1999 by a scribe from book copy #2. Book copy #2 is also a copy of the *Elements of Geometry*, and its content is explained by the fact that it was copied yesterday from book copy #1 by a scribe who then traveled through time to 1999 with book copy #2.
>
> Here, u_i and L_i are the same as in the *Infinite Scribes* case.

Again, this circular explanation is not complete because the specified behavior of the scribes (i.e., the laws) explains why both books have the same content, not why this content is that of the *Elements of Geometry* rather than, say, that of the Bible. And it fails to explain that because it leaves it open whether both books are copies of the *Elements of Geometry* or, say, the Bible. It does not determine that they are copies of the *Elements of Geometry*.

3.3 What About Well-Founded Explanations?

Of course, as Bliss and Priest rightly point out in another context, we might get an incomplete explanation in the well-founded (finite, non-circular) case as well:

> *Well-Founded Scribes* (Ordinary Case). Consider a series of n book copies. Book copy #1 is a copy of the *Elements of Geometry*; its content is explained by the fact that it was copied before by a scribe from an older book copy #2. This older book copy #2 is a copy of the *Elements of Geometry*; its content is explained by the fact that it was copied before by a scribe from a yet older book copy #3, and so on unto n.
>
> Here, u_i and L_i are the same as in the *Infinite Scribes* case.

The content of each book copy #i, where $i < n$, is fully explained in terms of its successor, but the content of book #n is left unexplained, so the explanation is incomplete. The case, in that respect, is exactly similar to that of the *Infinite Scribes*. Leibniz would have agreed. He believed, however, that there is a special item, namely, God, that is self-explanatory because He literally explains Himself. Others have argued that although they are not really self-explanatory, some items are "autonomous" in that they do not call for an explanation (Dasgupta 2016).

If there are autonomous or self-explanatory items, we might have a complete explanation in the finite, non-circular case. Just consider the *Well-Founded Scribes* (Extraordinary Case), which is exactly like the above ordinary case except that the book copy #n is very special: its content either literally explains itself or is at least autonomous.

I do not want to dispute that this toy example involving self-explanatory book content is implausible. I want to grant, however, that there might be plausible examples of the same form as this one. My claim is that despite Leibniz's contention, *some* infinite—and, more generally, non-wellfounded—explanations are complete.

4 Answering Leibniz's Objection

It is not difficult to modify our Leibnizian *Infinite Scribes* example to get a complete series. For a trivial case, consider what happens if we replace our faithful, regular scribes with *monomaniacal scribes*, i.e., scribes who, instead of making a faithful copy of the book they are given, always create a copy of the *Elements of Geometry*, whatever book copy they are given (they are so monomaniacal that they do so even if they are given no book at all).

In that case, the explanation does seem complete. The whole series, indeed, seems to be explained. Why? Because by explaining each transition (i.e., simply by mentioning the laws specifying the behavior of the scribes), one seems to explain why each book has the content it has. Unlike in Leibniz's *Infinite Scribes* case, what explains these transitions, namely, the behavior of monomaniacal scribes (the laws alone), does determine each item of the series. Actually, it necessitates them: necessarily, if all the scribes behave as specified, all books must be copies of the *Elements of Geometry*.

One might wonder whether this explanation really explains the existence (as opposed to the mere content) of the series of book copies (this is related to what we have called earlier the "extra-property objection"). Does it really

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explain the whole series of facts $(u_i)_i$, where u_i = 'book copy #i is the *Elements of Geometry*', or rather the series of conditional facts $(u'_i)_i$, where u'_i = 'if book copy #i exists, it is the *Elements of Geometry*'? The answer is that it really explains the whole series $(u_i)_i$ (and the implied existence claims) because we have specified in the laws that monomaniacal scribes always create a copy of the *Elements of Geometry*, whatever book copy they are given, *and do so even when they are given no book at all*. Had we not specified that in the laws, only the series of conditional truths would still be completely explained anyway.

The *Infinite Monomaniacal Scribes* is somehow trivial. There are more interesting examples of infinite chains of explanations that seem likewise complete. Consider:

> *Infinite Stick Adjusters.* Consider the infinite series of lengths $(l_i)_i$ of a given stick made out of a plastic lump. Given the number of molecules in the plastic lump, the length of the stick is bounded by b and B. Let l be a specific length between b and B. The length l_1 of the stick at stage #1 is explained by the fact that the stick has been adjusted before at stage #2 by a stick adjuster from a state in which it had length l_2, which length is explained by the fact that it has been adjusted earlier at stage #3 by another stick adjuster from a state in which it has length l_3, and so on to infinity..., where a stick adjuster is someone who takes a stick of length x and adjusts it so that its size becomes closer to a specific length l (where $b < l < B$):
>
> (i) If $l \leq x \leq B$, the stick adjuster compresses it in order to reduce its size by $\frac{x-l}{2}$ (so that its size becomes $x - \frac{x-l}{2}$).
> (ii) If $b \leq x \leq l$, the stick adjuster stretches it in order to augment its size by $\frac{l-x}{2}$ (so that its size becomes $x - \frac{x-l}{2}$).
>
> Here, L_i is the law that specifies the behavior of the stick adjuster at stage #i, and u_i is the fact 'if the stick exists at stage #i, it has length l_i'.

It takes little reflection to realize that necessarily if the stick adjusters behave as specified and if the stick exists at all, the stick will always be exactly l long. This can be deduced from the laws that specify the behavior of the stick adjusters alone. Intuitively, stick adjusters keep adjusting the stick to make its length closer and closer to l, and if you start with a finite stick, you will end

up, at the limit, with an l-long stick. But as each stick in the series is bounded and has infinitely many stick adjusters behind him, each stick will be l-long. More rigorously, the laws entail that $l_{i+j} = l + 2^j(l_i - l)$, so if for some i, l_i were different from l, the series $(l_{i+j})_{j\geq 1}$ would not be bounded, which is absurd by construction (unless otherwise mentioned indexes are natural numbers greater than or equal to one). This infinite explanation thus determines that the stick is always l-long—it does not leave the length of the sticks open. It accordingly seems to explain the whole series and, thus, everything there is to explain. It is arguably complete.[7]

We can come up with other, maybe simpler, examples of non-trivial complete infinite explanations. Consider the *Infinite Wheel Turners*.

> *Infinite Wheel Turners.* Take a wheel that is divided into four identical numbered sectors (respectively 0, 1, 2, 3). The sector s_1 on which the wheel has just landed is 1 because the wheel has just been turned by a first wheel turner at the beginning of stage #1 from a former sector s_2. It was in that sector because the wheel had been turned at the beginning of stage #2 by a second wheel turner from a former sector s_3..., where a wheel turner is someone who takes a wheel that has landed on sector x ($x \in \mathbb{Z}/4\mathbb{Z}$) and turns it so that it lands on sector $f(x) = 2x - 1$ ($f(x) \in \mathbb{Z}/4\mathbb{Z}$).
>
> Here, u_i is the fact that the wheel, if it exists, has landed on sector s_i at the end of stage #i. And L_i is the law specifying the behavior of the wheel turner #i (again, the laws are all identical).

As $f(0) = f(2) = 3$, $f(3) = 1$, and $f(1) = 1$,

- if the wheel lands on 1, it will always stay on 1 when it is turned again;
- and the wheel will always land on one (whatever the starting point), provided that it has been turned at least twice.

7 The reader worried by the "extra-property objection" regarding existence can check that we can get a version of the *Infinite Stick Adjusters* where u_is are uncontroversially existence-implying facts about the length of a stick rather than more modest truths that are conditional on the existence of the stick simply by stipulating that the stick adjusters *create* a copy of the stick they are given and then stretch it.

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So for all i, $s_i = 1$. This, moreover, holds necessarily, provided that the wheel turners act as specified. This explanation accordingly seems to explain everything there is to explain. It seems complete.

The reader can check that just as the Leibnizian incomplete *Infinite Scribes* series has a circular incomplete counterpart, all these examples of complete infinite explanations have circular counterparts that are complete (just add a time-travel twist to the stories).

Finally, there is an interesting contrast to be drawn between the *Infinite Stick Adjusters* case on the one hand and the *Infinite Wheel Turners* and the *Infinite Monomaniacal Scribes* on the other. Infinity or circularity (non-wellfoundedness) indeed seems somehow more important to the completeness of the explanation in the first case than in the two others. Indeed, the reader can check that in a simple well-founded version $(u_1, u_2, \dots, u_n)$ of the *Infinite Monomaniacal Scribes* case, the laws alone suffice to explain, if not the whole series, at least its $n-1$ first items $(u_1, u_2, \dots, u_{n-1})$. Roughly the same goes for the *Infinite Wheel Turners* if $n \geq 4$: the laws alone will suffice to explain the $n-3$ first items. In the *Infinite Stick Adjusters* case, however, unless our series of items is infinite or circular, even the length *of the first stick* will not be determined and explained. If the series is long enough, then at stage #1, the stick will necessarily be rather close to being l-long; its precise length will not, however, be determined by the laws. We can say that in the first case but not in the others, the completeness of our series of explanations is, so to speak, "entirely due to non-wellfoundedness."

Cases of complete, non-wellfounded explanations, and even more dramatically, cases in which the completeness is entirely due to non-wellfoundedness, show something very important, namely, that far from always being a liability, infinity and circularity can be explanatory productive and play an essential role in some explanations. We can draw an analogy here with proof theory. Despite a widespread assumption to the contrary, mathematicians do sometimes use circular or infinitely descending proofs in arithmetic. This is, for example, the most natural way to understand the so-called "proofs by infinite descent" (Fermat's proof of the irrationality of $\sqrt{2}$ is a classical example, and so is the classical proof of Euclid's division lemma[8]). Now, it can be shown that allowing such "non-wellfounded proofs" in Robinson Arithmetics yields

8 Euclid's division lemma states that for two integers a and b, with $b \neq 0$, there exist unique integers q and r such that $a = bq + r$ and $0 \leq r < b$.

classical Peano arithmetics (Simpson 2017).[9] In the guise of infinity and circularity, non-wellfoundedness is proof-theoretically productive. Cases like that of the *Infinite Stick Adjusters* show that what goes for proofs goes for explanations as well. Far from being an obstacle to good explanations, as suggested by the quotations of Fine and Schaffer,[10] infinity and circularity can do genuine explanatory work, but they will only do so in very specific cases. I would now like to find out what exactly distinguishes these cases.

5 Towards a General Case: Causal Explanations

All the examples of complete, non-wellfounded explanations we have given above imply *causal explanations*. Below, I will try to get more general results. I will first abstract general conditions on the completeness of causal explanations from the examples above and then show that the reasoning that yielded these conditions generalizes to metaphysical explanation and other "determination relations."

We can notice, first, that in all the examples above, the i-th item seems functionally determined by the $(i+1)$-th item. There is, in other words, a function f_i (depending on the law L_i) that accounts for the transition from the $(i+1)$-th item to the i-th item.[11] More precisely, in all of these cases,

- there is a parameter that can take different values at different stages (the content of the book, the size of the stick, etc.),
- such that the value of this parameter at stage $\#i$ is the result of applying the function f_i to the value of this parameter at stage $\#(i+1)$.

We can represent this functional dependence by introducing a series of functions $(X_i)_i$, where X_i associates to a possible world the value the $\#i$-th item of the series takes in this world and $\varnothing$ if the $\#i$-th item does not exist in this world. Let us also introduce the symbol $\circ$ for the composition of functions ($f \circ g$ is the function that associates $f(g(x))$ to x). Then, it seems that in all the cases we have envisioned so far,

9 I am indebted to Léon Probst for the discovery of this very interesting result and for the realization that proofs by infinite descent can be naturally interpreted as non-wellfounded proofs.

10 Schaffer and Fine are concerned with metaphysical explanations rather than causal explanations, but we will see that there are ground-theoretic analogs of the *Infinite Stick Adjusters* cases.

11 It should be emphasized that here, "function" is understood in the mathematical sense, where a function is just a relation R, such that if xRy and xRz, then $y = z$ (rather than as a causal role or as a trait selected by evolution for its causal role).

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- u_i has the form '$X_i(@) = x_i$' (where @ is the actual world and single quotes are a "fact formation device"), or in cases where the fact u_i is conditional on existence, 'Either $X_i(@) = \varnothing$ or $X_i(@) = x_i$'.
- There is a function f_i, such that $X_i = f \circ X_{i+1}$ (i.e., if the value of X_{i+1} in a world is a, then the value of X_i in this world is $b = f(a)$).

We can call f_i "the flow function" of the series.[12] In all of our examples above, $f_1 = f_2 = ... = f_i = ... = f$, and $L_1 = L_2 = ... = L_i = ... = L$, and we might say that the flow and the explanation are *uniform*. We could, however, construct explanations that are not uniform (say, by stipulating that some, but not all, scribes do not copy their book faithfully; see Billon 2023, 1942).

In the regular *Infinite Scribes* case, $f = f_{RS}$ is the identity function Id over book contents; in the *Infinite Monomaniacal Scribes* case, $f = f_{MS}$ is the constant function that associates the content of the book, which is the *Elements of Geometry*, to any content and even to the empty content of absent books. In the *Infinite Stick Adjusters* case, $f = f_{SA}$ associates the length $x - \frac{x-l}{2}$ to the length x; in the *Infinite Wheel Turners* case, $f = f_{WT}$ associates the sector $(2x - 1)[4]$ to the sector x...

In all the examples we have considered, there is also a natural metric associated with the values of our X_is. We can thus define a notion of distance and a notion of convergence on these values. I will argue that in cases such as these, the following conditions are both necessary for the non-wellfounded chain of explanation to be complete:

(CN1) f has a unique fixed point e (i.e., there is a unique value x of f, such that $x = f(x)$ and $x = e$).
(CN2) For all x, the series $(f^i(x))_i$ converges toward the same item e.

I will also argue that, conversely, the two following conditions are jointly sufficient:[13]

(CS1) f is contractive: there is $k < 1$, such that for all x, y, $|f(x) - f(y)| \leq k * |x - y|$.
(CS2) f is bounded.

12 I borrow the term from dynamical system theory, which should make sense by the end of the paper (section 9).

13 For these conditions to hold, we need to suppose that our metric space is "complete" (in the sense that every Cauchy sequence—intuitively, every sequence whose items can become arbitrarily close to each other—has a limit), which is unproblematic in all the examples we consider.

Intuitively, (CS1) means that f shrinks the space.

These conditions fit most examples above: (CN1) and (CN2) are only satisfied by the *Infinite Wheel Turners*, the *Infinite Monomaniacal Scribes*, and the *Infinite Stick Adjusters*; they are not satisfied in the *Circular Scribes* and *Infinite Scribes* cases. (CS1–CS2) are satisfied in the *Infinite Stick Adjusters* example but not in the *Infinite Wheel Turners* example (at least when $\mathbb{Z}/4\mathbb{Z}$ is fitted with the canonical metric, i.e., the distance between two points being the absolute value of their difference), which shows that (CS1–CS2) are not necessary.

By reflecting on an unbounded variant of the *Infinite Stick Adjusters* case, the reader can also check that (CS1) is insufficient by itself (i.e., without (CS2)) and that (CN1–CN2) are jointly insufficient. In that unbounded variant, *for any arbitrary length* l_1, we can construct a series of sticks such that stick #i is l_i-long and has been adjusted from stick #$(i+1)$ by one of our stick adjusters. Just take sticks such that $l_i = l + 2^{i-1}(l_1 - l)$. Accordingly, the fact that each item of a series is the length of a stick that has been adjusted by our next stick adjuster *does not determine the length* l_1 *of the first stick*, and it does not, a fortiori, determine the whole series of lengths. But if it does not determine it, it seems that it won't explain it either.

The argument for the necessary character of (CN1) and (CN2), and for the joint sufficient character of (CS1–CS2), involves two parts. It has a philosophical component first, connecting the notions of explanation and completeness to that of functional dependence and translating the claim that a non-wellfounded causal explanation is complete in mathematical terms. It also includes a mathematical component, demonstrating that the translated claim holds when (CS1–CS2) are satisfied and only holds when (CN1–CN2) are satisfied. The mathematical part of the argument is non-trivial, but it is philosophically uninteresting, so I will place it in the appendix. Now, I will slowly unfold the philosophical part of the argument, pausing at some interesting concepts that need to be introduced along the way.

5.1 *Insensitivity to Prior Items*

To say that a chain of explanations $(u_i)_{i\in I}$ is complete, as we have seen, is to say that it (fully) explains the whole series $(u_i)_{i\in I}$. In the non-wellfounded case, this means that by explaining the transitions from u_{i+1} to u_i, we fully explain the whole series $(u_i)_{i\in I}$. This, in turn, seems equivalent to saying

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that what explains the transitions from u_{i+1} to u_i (i.e., the laws $(L_i)_i$) fully zero-explains the whole series $(u_i)_{i \in I}$.

This means that in all cases of non-wellfounded, complete explanations, the laws $(L_i)_i$ alone will suffice to explain the first item u_1. Accordingly, the history $(u_2, u_3, ...)$ of the first item will be explanatorily irrelevant. Complete, non-wellfounded causal explanations will display a form of "historical irrelevance" or "insensitivity to prior items."

5.2 *The Explanation-Determination Condition*

In order to show that the Leibnizian infinite explanation is incomplete, we have argued that it does not *determine* the whole series. In order to show that the *Infinite Monomaniacal Scribes*, the *Infinite Stick Adjusters*, and the *Infinite Wheel Turners* are complete, we have argued that these chains of explanations do determine all the items of the series.

We have relied on the following explanation-determination conditions to the effect that the Leibnizian *Infinite Scribes* series fails to be complete because and *only because* it fails to determine all their terms:

(ED1) In order to be complete, chains of explanation, such as the Leibnizian *Infinite Scribes* series, need to determine all their terms.

(ED2) If a similar chain of explanations did determine all its terms, it would be complete.

I tackle (ED1) and (ED2) in turn.

(ED1) stems from the fact that (full) explanation is a *determination relation*, so that a (full) explanans (a final item) must, along with a law, *determine* its explanandum (an initial item). This is true for determinist explanations. One might worry this does not hold for non-determinist explanations, as found, e.g., in quantum mechanics. However, non-determinist explanations are arguably explanations in which the probability distribution of a variable (if not its effective value) *is* determined—this is what happens in quantum mechanics. So (ED1) is still arguably true in the non-determinist case, provided that we consider the explananda to be probability distributions.

Let us now move on to (ED2). It captures the idea that the only reason why the Leibnizian *Infinite Scribes* series is not complete is that it fails with regard to (ED1). Importantly, (ED2) does not imply that determination suffices for a full explanation: there are classical counterexamples to this claim, involving

asymmetry, overdetermination, or "pre-emption"; see Billon (2023, sec. 6). It only implies (and, in fact, it means) that if a series of explanations $(u_i)_i$ is such that the laws determine the full series, then the explanation is complete. And this claim is arguably true because when we talk about explanations of a series of items $(u_i)_i$ by laws, obstacles to the entailment from determination to explanation, such as asymmetry, pre-emption, and overdetermination, are not a real threat. The question of asymmetry does not even make sense in this context (the laws are not an explanandum here). As for the question of pre-emption and over-determination, they might make sense in cases where the laws are not uniform. Yet, if the laws determined the whole series but did not explain it because of pre-emption or overdetermination, a proper subset of the laws would arguably explain the whole series, and we would still have a complete explanation of the series.

Now, (ED1) and (ED2) entail that in our examples, the series we consider is complete iff (B) follows from (A):

(A) For all i, u_{i+1} (along with L_i) fully explains u_i.
(B) The laws $(L_i)_i$ alone determine the whole series $(u_i)_{i\in I}$.

Now, it is arguable that if there is something in (A) that can entail a determination condition such as (B), it is only the following determination condition that is entailed by (A):

(A*) For all i, u_{i+1} (along with L_i) determines u_i.

If that is so (and I will admit that it is), (B) follows from (A) iff it follows (B) from (A*). That is, iff

> Completeness Condition (First Version). That for all i, each item u_{i+1} (along with L_i) determines its antecedent u_i entails that the series of laws $(L_i)_i$ determines the series of items $(u_i)_{i\in I}$.

5.3 The Functional Account of Determination

One might wonder how we should analyze the sense of "determine" in the claim that explanation entails determination and in our first completeness condition. I must say it is very tempting to analyze it in terms of necessity (this is a temptation to which I have informally yielded a couple of times above, using modal considerations to assess determination claims). We might want

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to claim, for example, that an initial item determines a final item only if it necessitates it. This corresponds to what we might call the "strong functional account of determination." If U_i is a function that associates with a possible world the value the i-th item of the series takes in this world (U_i associates u_i to our world: $U_i(@) = u_i$), this account of determination says that

> STRONG FUNCTIONAL ACCOUNT OF DETERMINATION. The $(i+1)$-th item determines its antecedent (the i-th item) if there is a function g_i (depending on L_i) such that one of the following equivalent conditions is satisfied:
>
> (i) $U_i = g_i \circ U_{i+1}$.
> (ii) Necessarily, if the $(i+1)$-th item U_{i+1} is a in some world, then the i-th item U_i is $b = f(a)$ in that world.

Even though I believe that determination can indeed be understood as necessitation and that it is useful to think of it that way in what follows, it is not totally uncontroversial to do so, and it is not, strictly speaking, required. We can provide a broader account of determination below: the *weak functional account of determination*. It relies on a weakening of the conditional (ii) so that it becomes (a–b):

> WEAK FUNCTIONAL ACCOUNT OF DETERMINATION. The $(i+1)$-th item determines its antecedent (the i-th item) if there is a function g_i (depending on L_i) such that one of the following equivalent conditions is satisfied:
>
> (a) $g_i(u_{i+1}) = u_i$, and in close possible worlds where $U_{i+1} = u_{i+1}$, the value of U_i is still $g_i(u_{i+1}) = u_i$.
> (b) Had the (value of the) $(i+1)$-th item been slightly different because of a local miracle (say equal to u'_{i+1}), then the (value of the) i-th item would have been $u'_i = g_i(u'_{i+1})$.

The weak functional account of determination construes it not as necessitation but merely as a counterfactually supporting functional relation. Notice that (a) and (b) are equivalent to claiming that U_i and $g_i \circ U_{i+1}$ only coincide in a certain subset Ω of all possible words (a subset that contains the actual world and very close worlds), i.e., that $U_i|_\Omega = g_i \circ U_{i+1}|_\Omega$.

Why think that explanation must entail determination in this sense? Well, as far as causal explanations are concerned, good scientific explanations all seem underwritten by equations that yield, at least locally, a form of functional determination of this sort. Connectedly, the fact that causal explanations always yield a functional determination in this sense is entailed by the structural equation account of the "structural equations framework" of causation and causal explanation (Menzies and Beebee 2019), which precisely stems from scientific practice (Schaffer 2016). It is equally entailed by the more general "functional conception" of explanatory laws (Schaffer 2017).[14] More deeply, the claim that causal explanation requires such a "functional determination" stems from the fact that a cause must determine its effect and that an explanation follows a law. (a) and (b) are arguably the minimal conditions capturing these two facts.[15]

The reader who would not be convinced that either the conditions (ED1–ED2) or the functional account of determination universally hold should still grant that they hold rather generally (and, in particular, they hold in all the examples we have put forward until now and in those we will consider in what follows). This should be sufficient to maintain his interest in the conclusions of this paper.

14 The functional characterization of determination bears a strong resemblance to the structural equations framework of causation and the more general "functional conception" of explanatory laws (Schaffer 2017). There are important differences, though. First, my account is an account of determination, which I take to be a necessary condition of explanation, not an account of explanation itself. Moreover, as I understand them, both the structural equation framework and the functional conception of laws aim at accounting for the fact that the explained item *really depends* counterfactually on x (i.e., is sensitive to x), and so they require (at least) that the function f be non-constant. My "functional account of determination," on the other hand, is neutral regarding real counterfactual dependence and thus less demanding. It only aims at accounting for the fact that the explaining item determines the item it explains.

15 One might wonder why (b) and not just (a) is required to capture the idea that a cause determines its effect. (a) concerns the tokens u_{i+1} and u_i that happen to be the $(i+1)$-th item and the i-th item in our world and requires that the latter token be modally fixed by the former. (b) concerns the types represented by the functions U_{i+1} and U_i and requires that the latter be determined by the former. Now suppose that condition (a) held but not condition (b): imagine that had U_{i+1} been $u'_{i+1}(\neq u_i)$, then U_i would have been indeterminate (say that it could equally have been many different token items and that there is no fact of the matter regarding which it would have been). In such a case, one might still claim that the token u_{i+1} causally explains the item u_i, but it would be hard to maintain that this causal explanation follows laws.

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5.4 *A Mathematical Formulation of the Completeness Condition*

The distinction between the weak and the strong version of the determination condition is important philosophically but not very important formally. In what follows, I will, for the sake of simplicity, suppose that our function $(U_i)_i$ is only defined on Ω and accordingly omit the restriction and consider that U_{i+1} determines U_i iff $U_i = g_i \circ U_{i+1}$.

Now, interestingly, when, like in all of our examples, the items are facts u_i of the form '$X_i(@) = x_i$' or of the form 'Either $X_i(@) = \varnothing$ or $X_i = x_i$', it can be checked that the determination condition on $(u_i)_i$ (there is g_i such that $U_i = g_i \circ U_{i+1}$) is equivalent to the corresponding determination condition on $(x_i)_i$:

- There is f_i such that $X_i = f_i \circ X_{i+1}$,
- where f_i is exactly what we have called before the "flow function."

Using these conventions (with capital letters for functions other than f_i and f, and with the associated minuscules for the items that are their values), we can recapitulate:

- The item $\#(i+1)$ (along with L_i) determines the item $\#i$

means that

- there is a function f_i (that depends on L_i) such that $X_i = f_i \circ X_{i+1}$.

(In our examples, the explanations are uniform, so neither L_i nor f_i really depends on—i.e., is sensitive to—the index i.)

Similarly, to say that

- the laws $(L_i)_i$ determine each u_i in $(u_i)_{i \in I}$ *all by themselves*

means that

- there is a series $(E_i)_i$ of *constant functions* (i.e., functions whose output is insensitive to the input and so "depend on nothing") such that for all i, $X_i = E_i$.

Our chain of explanation is hence complete iff

> COMPLETENESS CONDITION (Second Version). 'For all i, $X_i = f_i \circ X_{i+1}$' implies that 'for all i, X_i is constant'.

As shown in appendix A, this second version of the COMPLETENESS CONDITION is all we need to get the mathematical running and show that (CS1–CS2) are jointly sufficient for completeness while (CN1) and (CN2) are both necessary.

6 From Causal Explanation to Metaphysical Explanation and Other Determination Relations

We have isolated general formal conditions (CN1–CN2) and (CS1–CS2) on the completeness of chains of causal explanations. They could be used to generate other examples of complete and incomplete such chains and to check whether current cosmology supports the idea that our universe might actually be explained by a complete non-wellfounded chain of causes.

Do the conditions (CN1–CN2) and (CS1–CS2) generalize to chains of metaphysical explanations? The answer is that they do. Why? Because in our reasoning, the fact that we were dealing with causal explanations, as opposed to some other relations, only intervened in our argument to the effect that causal explanations satisfy the explanation-determination conditions (and also, though only verbally, in our choice of dubbing the "insensitivity to prior items" in section 5.1, "historical irrelevance"). Yet if, as we have supposed, metaphysical explanations follow laws, they should functionally determine their explanandum as well: if the $(i + 1)$-th item fully grounds its antecedent, and if it does that according to a law, it should determine it in the required sense of a counterfactual-supporting functional dependence specified by (a–b) (see section 5.3, footnote 17, and also Schaffer 2017, who develops a couple of arguments to that effect). In other words, a metaphysical explanation must be a determination condition, at least in the weak functional sense isolated above. (ED1) should accordingly be satisfied. The same goes for (ED2) because the obstacles to the entailment from determination to explanation are arguably the same in the causal and in the metaphysical case, and they lose their grip when we consider the completeness question for chains of explanations (the problem of asymmetry does not arise in this context, and even if the laws determine without explaining the full series because of overdetermination or pre-emption, we would still have to say that a subset of the laws explains the whole series and that the explanation is complete).

More broadly, let us call an R-chain $(u_i)_i$ a chain of R-related facts of the form '$X_i(@) = x_i$', where R is a relation

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(I) whose logical form is $(x, L)Ry$, where x is an item or nothing ($\varnothing$), L a law that can be kept implicit, and y an item or a series of items;
(II) which is a determination relation in the sense that $u_{i+1}Ru_i$ entails that there is a flow function such that $X_i = f_i \circ X_{i+1}$.

Suppose that the R-chain is uniform in that the laws and the flow functions are always the same (for all i, $f_i = f$). Suppose, also, that we can define a metric on our items.[16]

Say, finally, that the R-chain $(u_i)_i$ is complete just in case

$$(\varnothing, L)R(u_i)_i,$$

and say that it is quasi-complete if the laws L alone determine the whole series. We can easily show that the conditions (CS1–CS2) and (CN1–CN2) are respectively sufficient and necessary conditions for the quasi-completeness of the R-chain. We can easily show that (CN1–CN2) are also necessary for completeness. Conversely, (CS1–CS2) will be sufficient for completeness when completeness is entailed by quasi-completeness.

As mentioned earlier, some philosophers believe that we should distinguish grounding from metaphysical explanation. Even if we have assumed that they were identical, these philosophers can still take R = grounding and get for grounding the exact same conclusions that we got for metaphysical explanations. Finally, this result might also apply to other determination relations, and in particular to R = supervenience (for supervenience, the questions of asymmetry preemption and overdetermination do not arise,[17] so the analog of (ED2) should trivially hold).

7 Non-wellfounded Chains of Grounds

Now that we have sufficient and necessary conditions for the completeness of non-wellfounded chains of grounds, we could try to use them to put forward "concrete" examples of complete and incomplete chains of grounds. As the conditions are formally similar to those that obtain in the case of causal explanations, we could also just try to adapt the examples we have already put forward. After all, if, as we have supposed, grounding is metaphysical

16 A metric, more precisely, that makes the space of items metric complete (in the mathematical sense of the term, see footnote 13).

17 This is precisely the reason why supervenience, which has long been used to capture something like metaphysical explanation, has largely been replaced by ground in this role.

explanation, the only relevant difference between a case in which an item x causally explains another y and a case in which x grounds y is that the laws regimenting the transition are natural, causal laws in the first case and metaphysical, grounding laws in the second. Maybe simply specifying that scribes, stick adjusters, and wheel turners are gods moved by metaphysical laws could do the trick?

7.1 *The Infinite Simulation and the Infinite Truth-Teller*

More convincingly, we could rely on the idea that the world contains various layers of reality that are grounded on each other but might closely resemble each other. This is an idea we can find in some interpretations of Plato (where forms resemble concrete reality, which resemble representations thereof…), but that is also popular among digitalists who believe that we might live in a simulation that is being run in an "upper" world that is itself being simulated in an "upper" world, etc. (Chalmers 2022). Some even suppose that this could go on indefinitely (Steinhart 2014). Of course, when A is a simulation of B, there is normally a causal story to tell: A has, for example, been programmed by someone to simulate B. Yet B is realized and grounded on A. Likewise, Plato famously provides (in the *Timaeus*) causal stories to explain the relationship between the Forms and the concrete objects we interact with. Yet these relations seem to involve grounds.

Now, we can obtain a ground-theoretic version of the *Infinite Scribes* that way if we imagine that our layer of reality is likewise grounded on another layer, which is itself grounded on another layer…, and that this series is infinite. In the example below, I adopt Chalmers (2022)'s theory of simulation, according to which a simulation of X is a digital object having the same causal structure as X so that a simulation of a simulation of X is still a simulation of X.

> *Infinite Simulation.* Layer #1 of reality contains just a digital object d_1, which has the same causal structure t as that of a small tree and simulates the latter. This simulation is realized (and grounded) in layer #2 on another object d_2, which is part of a computer of that layer. d_2 is realized and grounded in layer #3 on another object d_3, which is part of a computer at that layer… Let x_i be the causal structure of d_i. Here, u_i is the fact that the causal structure of the object at layer #i is t (with the same notations as above, $u_i =$ '$X_i(@) = t$').

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> The laws specify that each layer contains a simulated object realized in the next layer.

Here, the chain of ground is incomplete. Indeed, the fact that each object is a simulation of the next does not explain why our series is a simulation of a tree rather than one of (say) a bacteria. The reader can check that the flow function is the identity function over causal structures and has every causal structure as a fixed point. We have a simple example of an incomplete non-wellfounded chain of grounds.

The following *Infinite Truth-Teller*, which relies on truth-making rather than simulation/realization, is similar to the *Infinite Simulation* and to the *Infinite Scribes* case (the flow function is the identity over semantic values). It is an incomplete infinite chain of grounds as well.

> *Infinite Truth-Teller.* Let $(v_i)_i$ be a series of sentences, such that $v_i =$ 'v_{i+1} is true'. Let $(x_i)_i$ be the series of the truth-values of the v_is. Let u_i be the fact that the semantic value x_i of v_i is 1 ($u_i =$ '$X_i(@) = 1$'). u_1 is grounded on u_2, which is grounded on u_3, etc.

I find it harder to find an intuitively plausible ground-theoretic analog of our *Infinite Stick Adjusters* example using iterated simulations or infinite chains of sentences whose truths are grounded on each other.[18] Below, I argue that we can come up with interesting cases of *complete* and incomplete chains of grounds if we focus on the way facts about certain objects are grounded on facts about smaller objects (think about the way chemical facts are grounded in microphysical facts).

7.2 *Rep-Tiles and Fractals*

Chemists often use tilings by dominoes as models of the composition of solids. Facts about a solid modeled after a region of space can be considered as being grounded on facts about the arrangement of molecules (modeled after the

18 It might be possible to construct a truth-making analog of the *Infinite Stick Adjusters* using supervaluationist semantics. It might as well be possible to construct a "simulationist" analog of the *Infinite Stick Adjusters* by specifying that the degree of reality decreases geometrically with iterated simulations and by considering facts such as $u_i =$ 'the degree of reality at level i is zero'. I have not, however, been able to find simple and intuitively convincing examples of such analogs.

dominoes) tiling that region of space. More broadly, we can consider a world whose inhabitants are geometrical figures grounded on tilings thereof.

Below, I consider two such worlds. The first one involves rep-tiles. The second involves fractals. I did not find these by accident. Indeed, (CN1–CN2) imply that if u_1 is grounded on an infinite and complete chain of grounds, then u_1 can be obtained, at the limit, by the recursive iteration of the flow function f. This provides a nice recipe for examples of complete chains of grounds.

The first world is a rep-tile world. Rep-tiles are "self-replicating figures": figures whose copies can be assembled to produce a bigger figure with the exact same shape—figures that can, equivalently, be dissected into smaller copies of the same shape (see Gardner 2001, 46–58, and figure 1 for an illustration).[19] The second involves a fractal, i.e., a geometrical object whose structure is identical at every scale (we sometimes say that such an object is "self-similar").

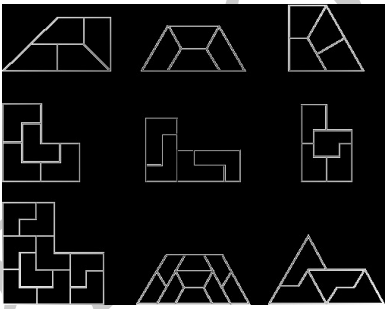

Figure 1: Examples of rep-tiles: the first two lines display rep-4 tiles, and the last line displays two rep-9 tiles and one rep-4 tile.

A Rep-Tiles World. Let us start with rep-tiles, then. We can divide rep-tiles according to the number of copies of themselves needed to make a bigger version of themselves. Here, we will focus on rep-4 tiles, that is, on figures that can compose bigger versions of themselves composed of *four* copies of themselves. Every triangle and every parallelogram is a rep-4 tile (they are not the only rep-4 tiles—see figure 1—but we will focus on these rep-4 tiles

19 A more complex, and probably more realistic, example involves a generalization of rep-tiles called "self-tiling tile sets" or "setisets" for short (Sallows 2014).

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to make things simpler). For every triangle and every parallelogram O, there is a unique rep-4 tile $f_r(O)$ made of O, three other copies of O, and such that O is on the bottom left corner of this rep-4 tile.

Conversely, for every triangle and every parallelogram O, there is a unique tiling (or "dissection") of O in four identical parts of the same shape as O, but with sides that are half the size of O's side. We can represent a tiling of O as a set of tiles (understood as compact regions, i.e., a bounded set of points that is topologically closed) whose union is O and whose intersection is reduced to the border of neighboring tiles.[20] We can label these tiles of O "O_a," "O_b," "O_c," and "O_d," using the left-to-right and up-to-down order.

Now, each of these tiles likewise admits a unique tiling in four similar parts. O_a is tiled by O_{aa}, O_{ab}, O_{ac}, O_{ad}; O_b is tiled by O_{ba}, O_{bb}, O_{bc}, O_{bd}, etc. We can call "i-iterated rep-4 tiling" a tiling obtained by i iterations of this operation (see figure 2).

The inverse of f_r is the function f_d, which is such that $f_d(O) = O_a$.

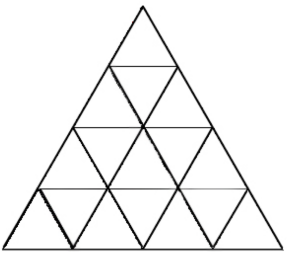

Figure 2: A triangle to which the dissection operation has been applied twice and that is accordingly tiled in 16 similar triangles.

Now, imagine a world that can only contain triangular or rectangular rep-4 tiles and in which each rep-4 tile is, at the next level, composed of its dissection in 4 tiles, whose tiles are in turn composed of their own dissections, etc. Consider the following series of rep-4 tiles indexed by levels. At level 1, the figure is an equilateral triangle $x_1 = abc$. At level 2, it is the smaller figure x_2 at the left bottom corner of x_1, such that x_1 is composed of three copies

20 We stipulate that figures are all compact and hence topologically closed, so the intersection of two bordering figures is non-empty, and we do not get a genuine partition of the original figure O. We could slightly modify the case to get a genuine partition, but that would make things uselessly more complex.

of x_2 ($x_2 = f_d(x_1)$). At level 3, it is the smaller figure x_3 at the left bottom corner of x_2, such that x_2 is composed of three copies of x_3 ($x_3 = f_d(x_2)$), etc. If we assume that facts about parts are explanatorily prior to facts about the whole they compose, the fact that the figure at level 1 is x_1 is grounded on the fact that level 2 is $x_2 = f_d(x_1)$.[21] This is grounded on the fact that at level 2, the figure at the bottom left corner is the yet smaller equilateral triangle $x_3 = f_d(x_2)$... The flow function here is f_r, the inverse function of f_d.

> *Infinite Rep-4 Tiles World.* Consider a world that contains a rep-4 tile at level 1 and, at each other level, a basic rep-4 tile on the bottom left corner of which the rep-4 tile at the preceding level is composed. At level #1, the figure is an equilateral triangle $x_1 = abc$, which (fact) is grounded in the fact that at level #2, the figure is its tile x_2 (where $x_2 = f_d(x_1)$ and $x_1 = f_r(x_2)$), which (fact) is grounded on the fact that at level #3, the figure is x_2's tile x_3 (with $x_3 = f_d(x_2)$ and $x_2 = f_r(x_3)$), etc. We have an infinite chain of grounds. Here, u_i is the fact that the figure at level i is x_i: $u_i =$ '$X_i(@) = x_i$', the flow function is f_r, and $X_i = f_r \circ X_{i+1}$. The metaphysical laws specify that the world contains only rep-4 tiles and that each rep-4 tile at level #i is composed of its tiling at level #$(i + 1)$.

Now, in this case, the fact that each item of this series of grounds is grounded on its successor according to the laws leaves it open whether they are all triangles or (say) squares (compare with the *Infinite Scribes* series). It also leaves completely open the size of the first item or its very existence. So, the series is not complete. In fact, it can be checked that the flow function f_r has no fixed point at all (it maps a figure to one of its proper parts), so the case does not satisfy (CN1).

A Fractal World. We can now move on to the fractal case. A dilation of factor x and center O is a function that regularly dilates the space of factor x around O. Such a dilation will, for example, transform a circle of center O and radius 1 meter into a circle of center O and radius x meters (if $x < 1$, the dilation will actually shrink the space).

21 Priority monists such as Schaffer (2010) believe that, on the contrary, facts about parts are grounded on facts concerning the wholes they compose. The reader can check that the example can be modified to suit priority monism: consider a world that contains iterated tilings (rather than iterated dissections) and replace f_d with f_r.

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Let abc be a filled equilateral triangle with 1 meter sides, and let f_{sa} be the dilation of center a and factor 1/2, f_{sb} be the dilation of center b and factor 1/2, and f_{sc} be the dilation of center c and factor 1/2. Consider the function $f_s = f_{sa} \cup f_{sb} \cup f_{sc}$. It is, so to speak, a "shrinking and duplication" function that associates to a figure O (understood as the shape of a compact set of points) three shrunk copies of it disposed at the extremities of the equilateral triangle. Now consider the figure s obtained at the limit by applying f_s iteratively to the filled equilateral triangle abc. This figure is called the Sierpiński gasket (or the Sierpiński sieve or the Sierpiński triangle) of corners a, b, and c.[22]

This way of generating the Sierpiński gasket might suggest a causal process (imagine someone repetitively shrinking triangles and assembling them…). Pace constructivists, however, we do not need to construe this way of generating the Sierpiński gasket (or indeed others) as really requiring some kind of diachronic construction. Moreover, even if constructivists were right to claim that the only good definition of s involves a causal or quasi-causal construction process, this causal construction story would be compatible with the following grounding claims concerning the output of this process. Indeed, by construction, the first figure $x_1 = s$ is composed of three shrunk copies (scale 1/2) of a second figure x_2 (take the one on the left bottom corner) such that $x_1 = f_s(x_2)$, x_2 is likewise composed of three shrunk copies (scale 1/2 again) of a third figure x_3 such that $x_2 = f_s(x_3)$, etc. If we assume, again, that facts about parts are explanatorily prior to facts concerning the whole they form, then the fact that the figure at level i in the series is x_i is grounded on the fact that the figure at level $i + 1$ is x_{i+1}, where $x_i = f_s(x_{i+1})$.

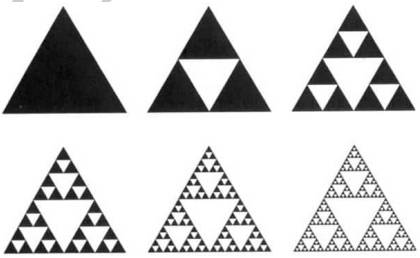

Figure 3: Sierpiński gasket obtained by iteration of f_s on a filled triangle.

22 This way of generating the Sierpinski sieve is called the "iterated functions system"; see, e.g., Falconer (2003, chap. IX).

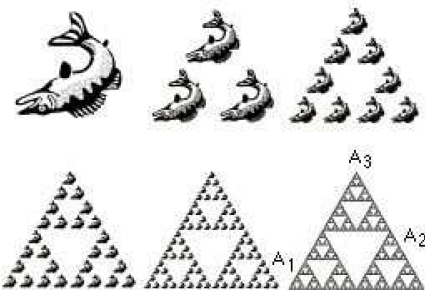

Figure 4: Sierpiński sieve obtained by iteration of a f_s on a fish… (from Barnsley, Hutchinson and Stenflo 2003).

3267 *Sierpiński Gasket World.* Consider a world that contains an infinity
3268 of levels. At level 1, there is a figure x_1, which is composed of three
3269 shrunk copies (scale 1/2) of the figure x_2 at level 2 ($x_1 = f_s(x_2)$).
3270 The figure at level 2 is itself composed of three shrunk copies (scale
3271 1/2) of the figure at level 3 ($x_2 = f_s(x_3)$)… The figure at level #1, x_1,
3272 is the Sierpiński gasket s, which (fact) is grounded on the fact that
3273 the figure at level #2, x_2, is s as well, which is grounded on the fact
3274 that the figure at level #3, x_3, is s as well, etc.

3275 Here, u_i is the fact that figure x_i at level #i is s: u_i = '$X_i(@) = s$'.
3276 The flow function is f_s, and $X_i = f_s \circ X_{i+1}$. The metaphysical laws
3277 state that there exists at least a figure (a compact set of points) and
3278 regiment the way (reflected by the flow function f_s) each figure is
3279 composed of three shrunk copies of the figure at the next level.

3280 Here, the laws alone determine both the shape of the figures in our series
3281 ($x_1 = x_2 = x_3 = \ldots = x_n = \ldots$ is the Sierpiński gasket s) and their existence.
3282 It thus determines the whole series u_i.

3283 Indeed, it can be shown that f_s is contractive,[23] and we can delimit our
3284 world so that it is bounded (we can specify that f_s is only defined on a bounded

23 The canonical Euclidian metric of the plane is not defined for figures (compact sets of points) but only for points, so in order to rigorously show that f_s is contractive, we need to introduce a distance on compact sets. This is typically done using Hausdorff distance (Falconer 2003).

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portion of space, including the triangle *abc*). This means that the flow function f_s satisfies (CS1) (contractive character) and (CS2) (boundedness).

The fact that our series is complete is also connected to a very peculiar property of f_s: whatever figure x we start with, the iteration of f_s on x will always yield the same figure. x can be a triangle (figure 3), a filled square, or even a fish (figure 4); the iteration of f_s on x will always yield the Sierpiński gasket s at the limit. The reader can check that this peculiar property is, in fact, equivalent to the satisfaction of (CN1) and (CN2).

Using (CN1), (CN2), and (CS1–CS2), we can construct other examples of complete and incomplete, non-wellfounded explanations. These might help us understand better what the difference between them amounts to. In appendix B, I put forward simpler (if less graphic), unidimensional versions of the above rep-tiles and fractal worlds: the *Zeno World* and the *Cantor Set World*.

7.3 *What About Circular Chains of Ground?*

Our examples of complete non-wellfounded chains of grounds involve infinite chains. Could we modify them, as we have modified the *Infinite Stick Adjusters* example, to put forward an example of a circular chain? Formally, this is not particularly problematic. The problem is rather to make metaphysical sense of the formal model—we have no simple ground-theoretic analog of time travel to make sense of circular metaphysical explanations.

Nolan (2018, 93) does try to make sense of something like a circular version of our *Sierpiński Gasket World* by describing a world in which "what appears to be our entire universe is just a sub-atomic particle in a larger universe, which is but a sub-atomic particle in a yet larger 'universe,' and so on," but where if you "go up through enough stages [...] you will arrive back at one of our sub-atomic particles." Even though I find the scenario conceivable myself, I must say that most people I have met—and a couple of referees for this journal—don't.

There are, in any case, simpler and less controversial (if less graphic) ways to construct complete circular chains of grounds. Consider:

No-Yes-Yes Sentences.

(1) "(2) is not true."
(2) "(1) is true, and (2) is true."

> The semantic value of (1) is grounded on that of (2), which is grounded on that of (1) and (2). So, we have a circular (non-uniform) chain of grounds. Here, u_1 is the fact that the semantic value of sentence (1) is true, and u_2 is the fact that the semantic value of sentence (2) is false (see below).

Classical logic and the naïve T-schema[24] show that (2) must be false and that (1) must be true. Indeed, if (2) is true, by one conditional T-out of the naïve T-schema, (1) is true, which means by T-out again that (2) is not true. This implies that (2) must be untrue and, by the other conditional T-in of the naïve T-schema, that (1) must be true. So the fact that (and the way) the semantic values of (1) and (2) are grounded on each other determines their semantic values.[25] We have an example of a *complete but circular chain of grounds*.

Note that we can put that reasoning in functional terms to match the other cases of grounding chains presented in this article. The semantic value [1] of (1) is such that $[1] = 1-[2]$, and $[2] = min([1], [2])$, so $[2] = min(1-[2], [2])$. The function that associates to a semantic value x, the semantic value $min(1-x, x)$, has only one fixed point, however, which is 0. This implies that (2) is false and (1) true. The fact that all orbits of the function $min(1-x, x)$ (all series of that result from the iteration of that function) converge to 0 moreover implies that the second version of the completeness condition is satisfied.[26]

24 The naive T-schema says that "p" is true entails p (T-out) and that p entails that "p" is true (T-in), where "p" is replaced by an arbitrary sentence. This naive T-schema is notorious for giving rise to semantic paradoxes when conjoined with classical logic and the existence of certain sentences, such as the liar-sentence "This sentence is false." One way to solve such paradoxes, once popular, consists in brutally restricting the naive T-schema to prevent self-referential truth-talk. Since the work of Kripke (1975), it is widely held that such an approach is too costly.

25 In other words, we are in a case in which condition (B) follows from (A).

26 This example is a non-paradoxical and non-hypodoxical variant of the truth-teller hypodox and the no-no paradox (cf. Billon 2019). The reader can check that this example can also be modified very simply to yield an *infinite* (and partly circular) complete chain of grounds:

(1) "(2) is not true."
(2) "(1) is true, and (3) is true."
(3) "(4) is not true."
(4) "(3) is true, and (5) is true."
(.) …

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8 Supercomplete Explanations and the Extra-Property Objection Again

In order to deny the significance of complete non-wellfounded chains of grounds, one might try to downplay the contrast between my examples of complete and incomplete chains of explanation cases by claiming that their comparison is not totally fair.

Consider, for instance, the contrast between my *Infinite Rep-4 Tiles World* case and my *Sierpiński Gasket World* case. For one thing, we could add "degrees of freedom" to the *Sierpiński Gasket World* case so that the chain of grounds becomes incomplete. Suppose, for example, that s is red but that our world allows for the possibility of blue and green figures. The color of s, unlike its shape, would not be determined by the infinite chain of grounds. The fact that s_1 was obtained by the iteration of f_s would indeed leave its color totally open. The latter would not be determined, and it would not be explained. Accordingly, the chain of ground would be incomplete.

Conversely, we could modify the *Infinite Rep-4 Tiles World* case so as to determine certain features that were not determined by our description of the case. For example, we might specify that our world contains only equilateral triangles and, accordingly, that f_d only ranges over such triangles. The fact that our initial figure is an equilateral triangle (but, it should be emphasized, not the size of this triangle) would thus be determined and explained by our infinite chain of grounds. Similarly, we could specify that there exists at least one figure in the world, as we did in the fractal case: the fact that there exists a figure (though not its shape) would then be determined and explained by our series.

The upshot is that in both examples, *the description of the case presupposes what can vary and needs to be grounded and what is fixed by the (more or less implicit) laws regimenting our example* (where these laws are understood broadly enough to include "structural features" of our cases, such as a specification of the possible entities it involves). But these presuppositions can be called into question, and what they presuppose (certain laws) might itself call for an explanation.

This is a fair point. In answer, we might concede that the morals of the fractal and rep-tiles examples are somehow modest. Indeed, it is only that:

- *some features* (here, for example, the shape)

- can be explained by *certain infinite or circular chains of grounds but not by others*,
- and that these features would not be explained by the corresponding finite, well-founded chains of ground (unless they start with an element that is self-grounded or, maybe, autonomous).

In other words, my conclusion would only be that *infinity or circularity (non-wellfoundedness) can do some explanatory work*. The fact that a figure s^* results from 12 iterations of f_s does not determine the shape of this figure or of its successors (it only determines that s^* will loosely resemble a Sierpiński gasket). The fact that it results from an infinity of iterations does determine its shape. Far from being an obstacle to good explanations, as suggested by the quotations of Fine and Schaffer, infinity and circularity can do genuine explanatory work, but they will only do so in cases where the completeness condition is satisfied.[27]

In any case, it should be emphasized that granting, as I have just done, that the right conclusion is just that non-wellfoundedness can do some explanatory work and that non-wellfounded explanations might only yield complete explanations in cases where the laws themselves are in need of explanation is

27 Morganti (2015) distinguishes between the transmission model and the emergence model of being and argues that the prejudice against infinite chains of grounds stems from a neglect of the emergence model:

> The "transmission model" of being, whereby the being of an entity at a given level of reality L_n is fully obtained, in a yes/no, all-or-nothing fashion, from the entity or entities at the immediately prior level L_{n-1}. [...]
>
> According to the emergence model of being, then, the metaphysical structure of priority and dependence gives rise to a dynamics analogous to that underlying the convergent [Zeno/geometric] series
>
> $$\sum_{n-1}^{\infty} \frac{1}{2^n}$$
>
> i.e., $1/2 + 1/4 + 1/8...$
>
> which converges towards 1 as n approaches infinity (and never becomes higher than 1). (Morganti 2015, 560–562)

However, if I understand him correctly, Morganti fails to draw the relevant distinction between cases where infinity does and where infinity does not do any explanatory work. Indeed, some examples of infinite chains of grounds seem to fit perfectly the emergence model but are incomplete nonetheless: this is the case, notably, of the *Infinite Rep-4 Tiles World* and of the *Zeno World*.

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not as concessive as it might seem. For one thing, many people believe that (what we consider as) the causal or even metaphysical laws of the world call for some explanations: fundamental physics tries to explain and unify the acknowledged laws, and some metaphysicians ask for grounds of grounds (Litland 2017). For another, once it is granted that infinity or circularity can do some explanatory work, one could start wondering whether the features that are not determined and explained by the chain of explanations itself and that we "hold fixed" by putting them in the laws L implicit in the description of the case (e.g., that shapes have no colors in the fractal case) could themselves be explained completely by another non-wellfounded chain of explanations. Who knows, the law L might be explained by some further law $\mathcal{L}_1$, which might be explained by $\mathcal{L}_2$, etc. And this chain of explanations might be complete. For sure, this chain of explanations will presuppose what we might call meta-laws ML, but they might likewise be explained by a complete, infinite chain of explanations… And once it is understood that infinite chains of explanations can be complete and can, more broadly, be explanatorily productive, infinite regresses should not scare us anymore—not even if we are looking for a complete explanation (where nothing but the laws call for explanation) or an ultimate explanation (where not even the laws call for explanation).[28]

9 Equilibrium Explanations and Essentialist Explanations

I have argued that non-wellfounded chains of explanations are somehow on a par with non-wellfounded chains of explanations with regard to completeness: the first can be complete if the flow function meets certain conditions (such as being bounded and contractive), and the second can be complete if they start with a self-explained item (in the sense of an item x such that x or a proper part of x fully explains x) or an autonomous item (in the sense that it does not call for an explanation). One might even think that they have a decisive advantage. Indeed, self-grounded or autonomous items, maybe in part because they have been often invoked by Theists, are sometimes considered

28 Let us call "weirdly" explained a fact that is explained in a case where $(u_i)_i$ is non-wellfounded but that would not be explained if $(u_i)_i$ were well-founded. As we have just seen, the fact that the first figure of the series s is a Sierpiński gasket inscribed in abc is weirdly explained in the fractal case, not the fact that there exists a figure. An interesting question is whether some facts cannot, because of their very nature, be weirdly explained by any kind of chain of explanations. If there were such facts, they could only be explained by being put explicitly in the laws. Many philosophers exposed to the arguments in this article have suggested that the fact that there exists something could not be weirdly explained.

spooky or supernatural, but complete, non-wellfounded explanations seem to be just as kosher as the bounded and contractive flow functions underlying them. In this section, I would like to show that complete, non-wellfounded explanations are indeed unproblematic by answering an important objection against that claim and showing that we already appeal to them ordinarily.

We have seen in section 5.1 that all complete, non-wellfounded explanations $(u_i)_i$ will display a form of "historical irrelevance" or "insensitivity to prior items": u_1 will not depend on the successors that explain it. (ED1) moreover entails that if our non-wellfounded chain of explanation is complete, the series $(u_i)_i$ is determined by the series of laws $(L_i)_i$. Assuming, as we have, that determination is functional, this, in turn, entails that if our chain of explanation is uniform (and for all i, $L_i = L_1$), then all x_i will all be equal to x_1, and all u_i will be of the form '$x_i = x_1$'. We can check that this is what happened in all our examples of complete, non-wellfounded chains of explanations (except for the *No-Yes-Yes Sentences*, which are not uniform).

Now, it might be wondered if such explanations, in which everything is determined by the laws alone rather than by prior items, are acceptable. I believe they are totally OK. One reason is that we do, in fact, commonly use such explanations. Below, I consider two rather common types of explanations that display "insensitivity to prior items" and in which the explanandum is determined by the laws alone: strict equilibrium explanations and essentialist explanations.

Consider, in the case of causal explanations, the so-called equilibrium explanations. The statistician Ronald Fisher explained why the sex ratio of males and females is approximately one by the fact that any deviation from this ratio would be progressively canceled by natural selection. This is a classical equilibrium explanation, and it displays, like our complete infinite and circular explanations, a form of historical irrelevance: to the extent that this explanation is correct, the sex ratio should always have been approximately one, and one can deduce that it is approximately one today without inquiring about its former values. Consider, to take another example, a lead ball in a closed bowl submitted to the law of gravity. One can explain why, after some time, the ball rests at the bottom of the bowl by the fact that it is the only equilibrium of the system.

More formally, an equilibrium explanation is an explanation of the state of a dynamical system, i.e., of a system whose state is described by a point Y in a geometrical space that depends functionally on a variable X, usually temporal: $Y = f(X)$. An equilibrium explanation explains the present state

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of the system by the fact that this state is an equilibrium of the system and that the present state is the result of the iteration of f on a given initial state x. The series $(f^i(x))_i$ is called the orbit of x. The equilibria of a dynamical system are determined by the explanatory laws. They are fixed points of f.

In many cases, equilibrium explanations are partial or elliptical. Sometimes, for example, we just state that the system is in state e because it is an equilibrium, but there are multiple equilibria of the system in which the system could end up being as well; or there is only one equilibrium e, but not all orbits $(f^i(x))$ converge toward e; or else all orbits converge towards e but some converge so slowly that the system could fail to be even close to the equilibrium, even after a huge amount of time. We can call "strict" an equilibrium explanation in which the system has only one equilibrium, e, and all orbits $(f^i(x))$ converge toward e, and "supers-strict," one in which f is bounded and contractive. A super-strict equilibrium explanation is, intuitively, a strict equilibrium explanation whose orbits converge very quickly (geometrically). It seems that a strict equilibrium explanation in which the prior states of the system are infinitely many is a full explanation of why the system is in the equilibrium state e. Moreover, such a strict equilibrium explanation is an explanation in which the prior states of the system are irrelevant: it is an explanation by the laws alone. In fact, the reader can check that the function f regimenting the dynamical system here is exactly what we have called the flow function before and that, formally speaking, all non-wellfounded, complete chains of explanations can be understood as such strict equilibrium explanations with an infinite number of prior states.[29] Even though classical equilibrium explanations are causal rather than metaphysical, we might thus consider complete infinite chains of grounds as a ground-theoretic version of equilibrium explanations.

Take essentialist explanations, now, such as the classical theist explanation that explains the existence of God by the fact that existence is part of His essence, or, lower on Earth, this essentialist explanation put forward by Kappes (2022, 444): the fact that either the sun is shining or it is not shining

29 There is a close connection between equilibrium explanations and optimality explanations, i.e., explanations, often found in biology or in certain interpretations of physics (cf. the Maupertuis-Leibniz interpretation of classical mechanics and (geometrical) optics), that explain the state of a system by the fact that it is in some sense optimal. This comes from the fact, exploited by optimization algorithms, that the optima of a (regular enough) function are the fixed point of a certain flow function and, in the good cases, the unique fixed point of a certain flow function towards which all orbits converge.

is explained by the essence of (classical) disjunction and negation. (Note that we do not need to commit to the precise essence of classical disjunction and negation to make the claim that they explain such a fact.)[30] Essentialist explanations are, or at least can be, perfectly kosher. It also seems that they can be understood, at least sometimes, as explanations by laws alone: in the classical logic example, we might say that the explanation relies on some laws of logic that are part of what define disjunction and negation, i.e., are essential to them.

We can conclude that there is at least one rather ordinary and unproblematic explanation by laws alone and that there might even be two: equilibrium explanations and essentialist explanations.[31]

10 How Simple Are Non-wellfounded, Complete Explanations?

In the introduction, I mentioned the fact that foes of non-wellfounded chains of grounds sometimes argue that they are explanatorily defective because they are incomplete. I have argued that they are wrong to suppose that non-wellfounded explanations need to be incomplete.

There is, however, another, a weaker version of the "explanatorily defective objection" against infinite chains of grounds. Instead of the principle of sufficient reason or one of its cognates, the latter invokes theory-choice considerations such as unity or simplicity and concludes that even though they are strictly speaking possible, infinite chains of grounds simply do not occur in the actual world. Thus, says Cameron (2008):

> It would be better to be able to give a common metaphysical explanation for every dependent entity [every item in the chain that is grounded on another one]. We can do that only if every dependent entity has its ultimate onto-logical basis in some collection of independent entities; so this provides reason to believe

30 Kappes (2022) calls explanations by laws alone "empty-base explanations" and provides many other interesting examples of such explanations.

31 An interesting question, which I will not have the time to address here, is whether some essentialist explanations can be analyzed as (ground-theoretic analogs of) equilibrium explanations. I raise this question because the connection between equilibrium explanations and optimality explanations noted in footnote 29 suggests a fascinating (if speculative) possibility, namely, that teleological essentialist explanations found in certain broadly Aristotelian or Leibnizian metaphysics might be underwritten by complete, non-wellfounded chains of explanations.

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> the intuition against infinite descent in metaphysical explanation. (Cameron 2008, 12)

Interestingly, the examples we have used to answer the stronger PSR-based objection against non-wellfounded chains of grounds allow us to dismiss this objection against infinite chains of grounds. For in all our examples of complete explanations, we have a simple explanation "for all dependent entities": it involves a simple structural feature of the chain of grounds, namely, the fact that its flow function f satisfies the second version of the completeness condition (see section 5.4), and in all but the *Infinite Wheel Turners* case, the even simpler fact that f is bounded and contractive.

It might not be trivial to compare two explanations for their theoretical virtues (a point rightly emphasized by Bliss and Priest 2018 in response to Cameron), but I think Cameron's point rests on the following comparison. Consider an ascending chain of grounds that starts with a foundational element v_1, which explains v_2, which explains v_3... Such a well-founded chain provides a simple explanation because, even if we have an infinity of items, the infinite chain can, so to speak, be factorized: v_1 explains all the following items. By contrast, suggests Cameron, a descending infinite chain such as the one we have considered, where u_1 is grounded on u_2, which is grounded on u_3, etc., cannot be factorized because there is no *ur*-item on which all the others are grounded. So, such a descending infinite explanation, concludes Cameron, must necessarily be complex. What is wrong with Cameron's argument is that he supposes that the only way to factorize or simplify an infinite (descending or ascending) chain of explanations involves a foundational item. This is wrong: in all the complete infinite cases we have considered, the descending infinite chains can be, so to speak, factorized if we invoke the fact that the laws (and, in the uniform cases, the law) suffice to explain all the items.[32]

32 In his latest book, Cameron (2022, chap. 3) grants that there *are* non-wellfounded chains of ontological dependence or grounds, but he argues that non-wellfounded chains of ontological dependence cannot be explanatory and that non-wellfounded chains of grounds are not normally explanatory. He relies, to that effect, on the quite unorthodox claim that grounding is not tied to metaphysical explanation. According to him, metaphysical explanation is indeed tied to understanding in a way that grounding is not. I do not have the room to discuss his view and his arguments here in any detail. I just want to mention that all my examples of infinite chains of grounds seem perfectly explanatory to me and that they do seem to provide a better understanding of the grounded items.

11 Conclusion

Most philosophers assume that non-wellfounded explanations are either impossible, non-existent, or, at least, incomplete or complex. Friends of non-wellfounded explanations usually accept that they cannot be complete but argue that this should not be counted against them. I have argued that non-wellfounded chains of explanations, be they causal or metaphysical, can be complete and simple, and, indeed, perfectly satisfying and *not* defective. The examples I have provided in support of that claim also show, I hope, that such explanations are also perfectly possible. Those who want a complete explanation of the world need not restrict their attention to foundationalist explanations starting with a self-explanatory or autonomous item. They can—in fact, they should—consider non-wellfounded explanations very seriously.

Appendices

Appendix A: From the Second Completeness Condition to (CS1–CS2) and (CN1–CN2)

Let us use the symbol "Im" for the image of a function. Let us also call:

$$\Pi_i = \mathrm{Im}\, f_i \cap \mathrm{Im}(f_i \circ f_{i+1}) \cap \mathrm{Im}(f_i \circ f_{i+1} \circ f_{i+2}) \cap \ldots \cap (f_i \circ f_{i+1} \circ f_{i+2} \circ \ldots \circ f_{i+j}) \cap \ldots .$$

Our completeness condition will be satisfied iff for all i, Π_i is reduced to a singleton whose member is the only value X_i can take.

It can be checked that if the flow is uniform ($L_i = L$ and $f_i = f$ do not depend on i),

$$\Pi_i = \mathrm{Im}\, f \cap \mathrm{Im}\, f^2 \cap \mathrm{Im}\, f^3 \cap \ldots \cap \mathrm{Im}\, f^j \ldots = \Pi$$

does not depend on i, and the completeness condition is simply that Π is a singleton.

> Completeness Condition (Uniform Case, Third Version). There is e such that
>
> $$\Pi = \mathrm{Im}\, f \cap \mathrm{Im}\, f^2 \cap \mathrm{Im}\, f^3 \cap \ldots \cap \mathrm{Im}\, f^j \ldots = \{e\}.$$

As for all i, $\mathrm{Im}\, f^{1+i} \subset \mathrm{Im}\, f$, this condition can be simplified:

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COMPLETENESS CONDITION (Uniform Case, Fourth Version). There is e such that Im f^i converges towards $\{e\}$.

We can now show that if the completeness condition is satisfied, (CN1) will be satisfied too. If $x \in \Pi$ and the completeness condition (take the uniform case, first version) is satisfied, there is y such that $x = f(y)$. But as $(x =)f(y) \in \Pi$ entails $y \in \Pi$ too, this means that if Π is a singleton and $x \in \Pi$, then x is a fixed point of f: $x = f(x)$. Conversely, if e is a fixed point of f, by the completeness condition (take the first version again), it belongs to Π. So if Π is a singleton, f has a unique fixed point e, and $\Pi = \{e\}$.

Similarly, it can be checked that if (CN2) failed, Im f^i could not converge toward $\{e\}$, and so the completeness condition (take the uniform case, second version) would not be satisfied.

Conversely, if f is bounded by m and contractive, it can be shown that $|f^n(x) - f^n(y)| \leq k^{n-1} * 2m$, which implies that f has a unique fixed point e and that all "orbits" $(f^i(x))_i$ converge towards e and, more importantly, that $\Pi = \{e\}$ (this is a variant of the Banach-Picard fixed point theorem). So, (CS1–CS2) are jointly sufficient for completeness.

Appendix B: The Zeno World and the Cantor Set World

Here is a unidimensional version of the *Infinite Rep-4 Tiles World*: the *Zeno World*. Let f_h be the dichotomic function that associates to an interval $[a, b]$ its first tile $[a, \frac{b-a}{2}]$. Let us call f_z its inverse. $x_1 = [a, b]$ is composed of two copies of $x_2 = f_h([a, b])$ $(= [a, \frac{b-a}{2}])$. Which is composed of two copies of $x_3 = f_h(x_2)$...

Zeno World. The fact that the interval at level i (two copies of which the interval at the preceding level is composed) is x_i is grounded on the fact that the interval at the level $i+1$ is x_{i+1}, where $x_{i+1} = f_h(x_i)$, that is, $x_i = f_z(x_{i+1})$. Here, u_i is the fact that the interval at level $\#i$ is x_i : $u_i =$ '$X_i(@) = x_i$', the flow function is f_z, and $X_i = f_z \circ X_{i+1}$.

Here again, the first fact is grounded on the second, which is likewise grounded on the third, etc., but that does not determine the first fact. It leaves completely open what the first interval is: it could very well be $[0, 2]$ or $[0, 17]$... More

deeply, the flow function f_z has no fixed point at all, so it does not even satisfy (CN1).

We can likewise put forward a simpler (albeit less graphic) one-dimensional version of the *Sierpiński Gasket World*. This one is known as the standard Cantor set. Let f_{ca} and f_{cb} be functions on a compact set of real numbers that associate to a set the image of this set by $g_{ca}(x) = \frac{x}{3}$ and $g_{cb} = \frac{2}{3} + \frac{x}{3}$, respectively. The Cantor set can be obtained by iteratively applying the shrinking (factor 1/3) and duplicating function $f_c = f_{ca} \cup f_{cb}$ to any compact set of real numbers. To fix the ideas, f_c associates $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$ to $[0, 1]$ and associates $\left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right]$ to $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$, etc.

> *Cantor Set World.* At level 1, there is a set x_1 that is composed of two shrunk copies (scale 1/3) of the set x_2 at level 2 (in the sense that $x_1 = f_c(x_2)$), the figure at level 2 is itself composed of two shrunk copies (scale 1/3) of the figure at level 3 (in the sense that $x_2 = f_c(x_3)$)... The figure at level #1, x_1, is the Cantor set s, which (fact) is grounded on the fact that the set at level #2 is x_2, which is grounded on the fact that the figure at level #3 is x_3, etc.
>
> Here, u_i is the fact that the figure at level #i is x_i : $u_i =$ '$X_i(@) = x_i$', the flow function is f_c, and $X_i = f_c \circ X_{i+1}$. The metaphysical laws state that there exists at least a compact set and regiment the way (reflected by the flow function f_c) each set is composed of two shrunk copies of the set at the next level.

Here again, (CS1) (contractive character) is satisfied, and we can define the domain of f_c so that (CS2) (boundedness) is satisfied as well. The infinite chain of grounds is complete.*

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Determination Relations and Metaphysical Explanations

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Ross Cameron (2022) argues that metaphysical infinitists should reject the generally accepted idea that metaphysical determination relations back metaphysical explanations. Otherwise, it won't be possible for them to come up with successful explanations for the existence of dependent entities in non-wellfounded chains of dependence. I argue that his argument suffers from what he calls the finitist dogma, although indirectly so. However, there is a better way of motivating Cameron's conclusion. Assuming Cameron's principle of Essence, explanations for the existence of dependent entities turn out to be circular if determination relations back explanations. This latter argument provides a stronger case as it puts the foundationalist under significant pressure, besides putting the infinitist under some pressure, to deny the idea that determination relations back explanations.

Reality is vast and variegated. There are concrete objects, events, mathematical objects, persons, economies, etc. This variety, nevertheless, doesn't preclude unity in what exists. Things are connected to each other via different kinds of relations, making it possible for us to comprehend them in unison. A decrease in the interest rate causes an increase in inflation; a cell is composed of organelles; the fact that my pen is cylindrical grounds the fact that it has a shape; the set of natural numbers ontologically depends on the existence of natural numbers, etc. Some of these relations are causal and are relied on in causal explanations. Others are non-causal. *Metaphysical determination relations* has lately been used in the literature as an umbrella term for the other, non-causal kinds of relations.

It seems obvious that causal relations are explanatory. Scientists, for instance, appeal to causal relations in their explanations, and we think that

those explanations are supported by relevant causal relations. It is said that one explains a phenomenon by determining its cause (Ratner 2003).[1]

Can we say the same thing about non-causal determination relations as well? Is it obviously true that metaphysical determination relations support explanations? Some think that it is difficult to see what use metaphysical determination relations are for unless they are explanatory (Bliss 2018). Based on this motivation, metaphysical determination relations are usually taken to support explanations of a distinct kind, i.e., *metaphysical explanations*. Just like causal relations back causal explanations, metaphysical determination relations back metaphysical explanations. One way to understand metaphysical explanations is to take them as *what it is* claims (Cameron 2022, 135). Consider the relation of set membership. What it is for the singleton $\{x\}$, for instance, to exist is to have x as an only member. Now, the idea is that this explanation is backed by a relation between the singleton $\{x\}$ and its member x.

Ross Cameron (2022) attacks this widespread assumption. Unless you are a metaphysical foundationalist—that is, you think a chain of metaphysical determination should be wellfounded, i.e., tethered to an ultimate foundation—you need to accept that there are certain cases where metaphysical determination relations don't back metaphysical explanations. This argument, if successful, forces metaphysical infinitists and metaphysical holists—those who think, respectively, that infinite or circular chains of metaphysical determination untethered to a foundation are possible—to deny that metaphysical determination relations necessarily back metaphysical explanations.

The layout of the essay is as follows: After summarizing Cameron's argument in section 1, I criticize it in section 2 by appealing to a distinction between *Objectivist* and *Subjectivist* senses of explanation and to the *Hume-Edwards Principle*. In section 3, I offer an alternative to Cameron's argument by using Cameron's premises. Throughout 3.1–3.3, I evaluate ways of reacting to the argument I offer. I conclude in section 4 by signifying the advantages of my alternative over Cameron's argument.

1 Cameron's Argument

Cameron's argument is based on two principles:

1 Thanks to an anonymous referee for suggesting this analogy.

ESSENCE. When x ontologically depends on y, the fact that x ontologically depends on y is part of x's nature.

DEPENDENCE. When x ontologically depends on y, this dependence holds at least partly in virtue of the existence or nature of y.

Applied to our earlier example, ESSENCE says that fact $<\{x\}$ depends on $x>$[2] is part of $\{x\}$'s nature. $\{x\}$ is the singleton of x, and hence a complete explanation of its nature has to include an appeal to the fact that $\{x\}$ depends on x; $\{x\}$ could not exist and be the set it is without depending on x. And DEPENDENCE says that x is at least partly responsible for the fact that $\{x\}$ depends on x. Either x and $\{x\}$ are responsible for the dependence relation together or it holds solely because of x. In either case, x is at least partly responsible for the relation.

Now, these two principles make the success of any explanation for the existence and nature of $\{x\}$ hostage to any explanation for the existence and nature of x. The mechanics of this argument are as follows: Because $<\{x\}$ depends on $x>$ is part of $\{x\}$'s nature, by ESSENCE, we need to appeal to this dependence relation in order to give a complete explanation for $\{x\}$. Because x is at least partly responsible for the dependence relation, by DEPENDENCE, our appeal to the dependence relation takes us to x. Thus, in order to give a complete explanation for $\{x\}$, we are compelled to account for x. In other words, the success of our explanation for the existence and nature of $\{x\}$ is hostage to the explanation for the existence and nature of x.

Now, if x is not ontologically dependent on another entity, it will be possible to provide a successful explanation for its existence and nature. This successful explanation will form the bedrock for the success of the explanation for $\{x\}$. But if x is a dependent entity, we will be faced with the same problem once again. For instance, if x is a singleton, whose sole member is y, the success of the explanation for x will be hostage to the explanation for y. For, again, ESSENCE will take us from the explanation for the existence and nature of x to the fact that x depends on y, and DEPENDENCE will take us from there to y. The success of the explanation will be deferred once more. Moreover, unless this ontological dependence chain bottoms out at the level of a foundational entity, this deferral will go on and on. If the chain is infinite or circular, the

2 Throughout the paper, I will use '$<f>$' to mean 'the fact that f' following common practice.

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success of the first explanation, along with any other explanation in the chain, will be deferred infinitely.

This means that in order for us to have a complete explanation for the existence and nature of an entity on a chain of ontological dependence, that chain needs to be tethered to a foundation. Even if there is no problem about the possibility of infinite or circular non-wellfounded chains of dependence per se, metaphysical foundationalism will seem to have an explanatory advantage over its rivals. Therefore, Cameron concludes, it's better for the metaphysical infinitist and holist to deny the claim that metaphysical explanations are backed by metaphysical determination relations, thereby defusing the supposed explanatory disadvantage.

2 Completing an Infinite Chain of Explanation

It isn't clear whether Cameron's argument, by itself, can motivate this conclusion, for it depends on a questionable further assumption: that an infinite chain of explanation cannot be successfully completed. The reason for this is not merely that the explanatory chain is infinite. For, that would be begging the question against infinitism. The reason why an infinite chain of explanation cannot be successfully completed, he says, is that one cannot even make a single successful explanation in such a chain. The success of a single explanation in the chain is hostage to the successes of every subsequent explanation in the infinite chain; since the chain is infinite, the success of the "original explanation is never established" (Cameron 2022, 100–101).

2.1 *Subjectivist vs. Objectivist Explanations*

Yet, unless we are misled by the spatiotemporal connotations of the chain metaphor, there is no reason to think this. When we talk about explanation in this context, we are not using what Bird (2005) calls the *subjectivist* sense of the word but the *objectivist* sense of it. In the first sense, an explanation is an act of explaining. As with all other doings of a subject, an explanatory act takes time. As Lewis (1986) points out, alluding to Bromberger (1965), questions like "Who gave the explanation?" "When was the explanation given?" or "Where was it?" apply to this sense of the word.

In some contexts, however, explanation seems independent of subjects. For instance, when the physicalist says that there is a physical explanation for every physical phenomenon, the claim is not that each physical phenomenon

is explained by some person or that a complete physical explanation of every phenomenon could be given by many people working in tandem. Rather, what is claimed is that each physical phenomenon stands in an explanatory relation to another. Therefore, the questions cited above do not apply to this second sense. Instead, questions like "Is it very complicated?" "Who thought of it first?" or "Does anyone know it?" apply.

Explanations in this objectivist sense are not doings of subjects but propositions in which explanatory relations are featured. These are the relations that allow us to make sense of the world. Any act of explanation is supposed to allude to these explanatory relations. Nevertheless, it is possible for the objectivist explanation not to be conveyed at all in such an explanatory act. Lewis states that an explanation in this second sense "might even be information that never could be conveyed, for it might have no finite expression in any language we ever use" (1986, 218).

Now, it would be sensible to deny the possibility of a complete infinite chain of subjectivist explanation, since such a claim would require infinitely many acts of explanation. When, however, infinitists claim that an infinite chain of explanation can be complete, they aren't positing infinite chains of explanatory acts. Rather, they are using the objectivist sense of explanation. And in this sense, as Lewis endorsed above, the explanatory story can in fact be infinite, even though it might be impossible to express it in an explanatory act.

2. *Completing an Infinite Chain of Explanations*

How, then, should we conceive an infinite chain of explanation as successfully completed? The notion of completeness we need to employ here, I claim, needs to address completeness requirements in holist and infinitist cases as much as foundationalist cases, thereby staying impartial in the debate among these camps. The following criterion, namely the *Hume-Edwards Principle*, meets this condition: An explanatory chain is complete if and only if each and every explanandum in the chain gets an explanation (Rowe 1970).[3] That is to say, if nothing in the chain is left out, we can say that it is a complete chain of explanation. Applied to the case of infinite chains, this means that an infinite chain of explanation is complete provided that every entity in the chain gets an explanation in the objectivist sense.

3 I need to thank an anonymous referee for their helpful suggestion in contextualizing this principle.

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What reason can one have to deny that every entity in an infinite chain can get an explanation? If the success of each explanation were hostage to the existence of a further explanation in the subjectivist sense, this would require us to try to complete a never-ending journey. However, when we formulate the explanation in the objectivist sense, we don't have such a problem. The success of an explanation, in this case, won't be hostage to the existence of an explanatory act but to the existence of the next explanation in the objectivist sense, which doesn't require time.

Pruss (1998) provides three counterexamples to the Hume-Edwards Principle. I will now go over Pruss' examples and show that they fail.

In the first case, there is a cannonball that is shot out at 11:58 and lands at noon. Let C be the set of time-slices of the cannonball's movement beginning from but not including 11:59 to and including 12:00. That is, C is the collection of the cannonball's states in the last minute of its movement. Pruss argues that C fits the Hume-Edwards Principle. It consists of infinitely many states at infinitely many time-slices in the last minute. Moreover, given the relevant Newtonian laws of physics, each state within C can be completely explained by a state preceding that state in C. Now, if the principle is true, then C must be completely explained by itself, since each and every state in it is allegedly explained by a previous state. This would mean that the movement of the cannonball in the last minute could be explained without appealing to the cannon at all, which is absurd. Therefore, the thought goes, the principle is false.

This case, on the face of it, seems to fit the Hume-Edwards Principle. Because the time-slice at 11:59 is not included in C, there is no time-slice in C that can be deemed the first in the series. So, whichever time-slice you pick, there will be infinitely many time-slices preceding it in C. So, we are expected to accept that each explanandum in C is explained by its predecessor in C.

Nevertheless, I claim, this counterexample doesn't work. For, either the state of the cannonball at 11:58, call it T^*, doesn't explain anything in C or C is not complete and hence doesn't fit the Hume-Edwards Principle. But surely, T^* explains infinitely many states succeeding it in C. For, according to Pruss' account, any state of the cannonball is explained by states preceding it, and there are states in C that are preceded by T^*. The fact that we aren't able to pick a single state in C that could be deemed *the immediate successor of* T^* doesn't prevent us from stating that there are states in C succeeding T^*. There are, in fact, infinitely many successors of T^* in C. Therefore, there are infinitely many explananda in C that are left without an explanation when T^*

is not included in the explanation. It's true that *C* is not self-explanatory, but it's not true that every member in *C* is explained. Therefore, Pruss' cannonball example doesn't meet the Hume-Edwards Principle and, hence, doesn't falsify it.

This is a paradigmatic example of using a mistaken analogy against non-wellfounded infinite chains. Non-wellfounded infinite chains are, as Oberle (2022) points out, categorically different from wellfounded infinite chains whose foundation is cut out. An infinite series of causation whose primary cause is cut out is not a non-wellfounded series. For, a non-wellfounded series wouldn't need a primary cause in the beginning. *C* in Pruss' example is like a wellfounded series whose link to its foundation is severed. The fact that we can slice *C* into infinitely many states doesn't automatically make it a non-wellfounded series. That's why we can coherently claim that without T^*, there are states in *C* that aren't explained. If *C* was a non-wellfounded infinite series, we wouldn't be able to claim that there are states outside *C*, like T^*, without which we cannot explain certain states in *C*. So, even though Pruss' target wasn't non-wellfounded infinite chains but the Hume-Edwards Principle, his argument requires finding such a non-wellfounded infinite chain and demonstrating that explaining all the explananda in it doesn't suffice to explain the whole of the chain.

Pruss' remaining two counterexamples target circular explanations, and they reiterate the same mistake of confounding non-wellfounded chains with wellfounded chains without a foundation. In one of them, there is a classical time travel scenario in which a woman has traveled back in time to give birth to herself. In the other, we are expected to think that the collection of all the chickens and the collection of all the eggs in the world form an explanatory cycle. Both examples fail to address the issue of completeness.

In the time travel example, either the time travel circle exists independently of anything else or it depends on an external cause. If it is independent, then it can be claimed that explaining the parts is sufficient to explain the whole of the circle. This doesn't even count as biting the bullet since the bullet was bit when you accepted that such an independent circle of time travel is possible. The explanatory story simply would follow the ontological story. If, however, you think that the time travel scenario wouldn't be metaphysically possible without an external ground, then the parts of the circle are not completely grounded on each other since they also depend on the external cause as well. So, without an appeal to the external cause, there will be explananda in the

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chain that aren't explained. In either case, the Hume-Edwards Principle is not falsified.

The last example is analogous to the second horn of the time travel example. The circle formed by the set of all the chickens and the set of all the eggs does depend on external causes. Given the evolutionary history of chickens, the set of chickens and the set of eggs don't form a closed circle. Therefore, an explanation of those two sets would not leave any unexplained explananda in the circle. Thus, the principle will again not be falsified.

To sum up, the Hume-Edwards Principle provides a criterion of completeness favorable to the infinitist.[4] It allows her to say that every link in an infinite chain gets an objective explanation. Although Cameron's argument doesn't directly depend on finitist dogma since it doesn't depend simply on the impossibility of infinite chains of explanations, it still reiterates a similar mistake at the level of success of explanations. It depends on the idea that an infinite chain of explanations in which the success of each is hostage to the success of the next cannot be complete and that it cannot be complete because it is infinite. Therefore, as it stands, Cameron's argument is unsuccessful in forcing the finitist or holist to deny that metaphysical explanations are backed by metaphysical dependence relations.

3 The Circularity Argument

There is, however, a better way of arguing that explanations should be pulled apart from determination relations. Cameron's argument was supposed to compel the infinitist to separate metaphysical explanations from metaphysical determination relations. As we will see, my argument *could* be used to motivate the infinitist to separate explanations from determination relations: she can only dodge this pressure on pain of losing an epistemic advantage over metaphysical holism. Metaphysical foundationalists, however, will be under more significant pressure to separate explanations from determination.

Assuming the principle of ESSENCE, the idea that explanations follow from determination relations faces a circularity problem. For, the dependence

4 See Billon (2023) for a different conception of how non-wellfounded infinite chains of explanations can be complete. Billon doesn't take the Hume-Edwards Principle to be providing a successful account of completeness for non-wellfounded infinite chains since he thinks this principle fails to give a separate explanation for the existence of the whole chain. However, he proposes another account according to which there are complete non-wellfounded infinite chains of explanation.

relation is both part of what is supposed to give rise to the explanation for the existence of the dependent entity and part of what is to be explained.

Part of the essence of a set is the fact that it depends on its members for its existence and identity. The fact that $\{x\}$ depends on x for its existence and identity, for instance, is part of the essence of $\{x\}$. So, any explanation for the existence and nature of $\{x\}$ also needs to account for the fact that $\{x\}$ depends on x. Yet, given that explanations are backed by determination relations, the fact that $\{x\}$ depends on x is among the things that make the explanation possible. Hence, the fact that $\{x\}$ depends on x is part of both *explananda* and *explanantia*. Therefore, the explanation is circular.

The circularity argument can be formalized as follows:[5]

(1) If x depends on y, then $<x$ depends on $y>$ is part of x's nature. (Essence)
(2) $\{x\}$ depends on x. (Premise)
(3) $\therefore <\{x\}$ depends on $x>$ is part of $\{x\}$'s nature. (By 1 & 2)
(4) If a fact f is part of x's nature, then f must be among the *explananda* of any explanation of x's existence and nature. (Premise)
(5) $\therefore <\{x\}$ depends on $x>$ must be among the *explananda* of any explanation of $\{x\}$'s existence and nature. (By 3 & 4)
(6) If x depends on y, then $<x$ depends on $y>$ must be among the *explanans* of any explanation of x's existence and nature. (Backing)
(7) $\therefore <\{x\}$ depends on $x>$ must be among the *explanans* of any explanation of $\{x\}$'s existence and nature. (By 2 & 6)
(8) $\therefore <\{x\}$ depends on $x>$ must be among both the *explananda* and the *explanans* of any explanation of $\{x\}$'s existence and nature. (By 5 & 7)

Assuming Backing and Essence, we have arrived at the conclusion that explanations regarding the existence of certain dependent entities are circular. How should we react to this conclusion? There are three main options. Rejecting Essence, embracing circularity, or rejecting Backing. I will now go over these options one by one.

5 I am indebted to Jonathan Payton for the fruitful discussion we had and the constructive criticism he offered regarding this formalization.

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3.1 *Rejecting Essence*

One way to oppose the circularity argument is to reject ESSENCE. ESSENCE, after all, is a strong condition requiring dependent entities to have their dependence relations essentially. This might not obtain in all cases of metaphysical determination. In the case of a ship and the parts composing it, for instance, the ship ontologically depends on its parts, but this dependence isn't part of the nature of that ship. That ship can be that very ship, even if it has different parts. Then, the circularity in the explanations for dependent entities can be avoided in cases involving certain metaphysical determination relations like composition.

It's true that there are cases where ESSENCE doesn't apply. But, still, there are also cases where it applies. And, provided that we don't embrace circularity, that there are some cases to which ESSENCE applies is sufficient for the project of pulling apart explanations from determination relations. For, as Cameron (2022, 99) states, that project requires showing not that explanations can always be separated from determination relations but that they can be separated at least in some cases of determination.

What remains, then, is demonstrating cases where ESSENCE applies. And, incidentally, we have just the case for that: part of the essence of a set is the fact that it depends on its members. It would be impossible to know what a set is and to be able to identify which set one is thinking and talking about without grasping the idea that sets depend essentially on their members (Lowe 2016).

Let's try to make sense of the scenario in which we reject ESSENCE. What would this mean for sets? Applied to our example, ESSENCE states that part of what it is to be $\{x\}$ is the fact that $\{x\}$ depends on its member, x, for its existence and identity. To reject this, one needs to claim that $\{x\}$ could have been what it is without depending on x for its identity. But how can $\{x\}$ be what it is, i.e., the singleton of x, without depending on x? Is it possible for $\{x\}$ to be the singleton of x if it has another entity, say y, as its member? This is obviously absurd. Then, at least in the case of sets, we can say that ESSENCE is applicable.

3.2 *Embracing Circularity*

Another way to react to the circularity argument is to accept the conclusion but render it harmless by opening a place for circular metaphysical explanations.

If we embrace circularity in metaphysical explanations, the argument won't compel us to deny that metaphysical explanations are backed by metaphysical determination relations. But holists, infinitists, and foundationalists aren't in the same boat in the debate concerning circularity.

First of all, metaphysical holists already embrace circularity in metaphysical structures. They take circular chains of metaphysical determination relations to be possible. This means that regress, by itself, isn't a problem for them. Why not, then, embrace both the backing claim and circularity in this matter, and claim that metaphysical explanations, following metaphysical determination relations, form circles?

Circular explanations are usually rejected on the basis of epistemic concerns. For instance, a subject wouldn't gain any knowledge from a circular explanation. Even if there isn't any problem about circularity in general, there is, the objection goes, a problem with circularity in explanations.

In response to this objection, we need to recall the distinction I employed above between explanations that are dependent on subjects and explanations that aren't. In the sense we use when we talk about metaphysical explanations, explanations are independent of subjects. That is the reason why I claimed that an infinitist can accept the possibility of infinitely long chains of metaphysical explanation, even though no human being can get a complete grasp of it, let alone give a complete account of it. This is to say that epistemic concerns relevant for explanations in the subjectivist sense aren't necessarily relevant for explanations in the objectivist sense. Metaphysical holists, then, can bite the bullet and claim that circularity is acceptable in metaphysical explanations as it is acceptable for them in metaphysical determination relations. Metaphysical holists, therefore, are under no pressure to reject backing on the face of the circularity argument, for embracing circularity is consistent with their overall stance in this debate.

Metaphysical infinitists have two options. They can either side with holists and embrace circularity, which will enable them to hold on to Backing, or they can reject circularity, in which case they will be under pressure to deny Backing.

Embracing circularity is arguably consistent with metaphysical infinitism in general. The account of completeness in explanatory chains I employed above, for instance, allows for both infinite and circular complete chains of

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metaphysical explanations.[6] If everything to be explained in a chain gets an explanation, that explanatory chain is complete regardless of whether it is infinite or circular. If an infinitist chooses this path, she will be under no pressure from the circularity argument to deny BACKING.

Metaphysical infinitists, nevertheless, can stay cautious about circularity on the basis of epistemic concerns. Thus, they can claim an advantage over holism despite being in the same boat in arguing for the possibility of non-wellfounded chains of being. An example of such an argument might be as follows: Even though it won't be possible for a subject to grasp an infinite chain of metaphysical explanations completely, every act of explaining that subject attempts has the potential to appeal to further links in the chain of explanation and, hence, to yield new knowledge. Therefore, metaphysical infinitism meets epistemic requirements. However, the same cannot be said for metaphysical holism. Once a subject grasps a circular chain of metaphysical explanations in its totality, their acts of explaining will stop yielding new knowledge. For, none of the *explanans* featuring in their acts of explanations will be new. Therefore, metaphysical infinitism has an advantage over metaphysical holism. Should metaphysical infinitists choose this latter path and hold on to their epistemic advantage over metaphysical holism, they will be under pressure from the circularity argument to deny BACKING.

Lastly, metaphysical foundationalists have only one viable option regarding circularity. They must reject circularity, for they do believe that there is a problem with circularity in general. Otherwise, the foundationalist case against holism would be weakened since it is raised mainly on the basis of circularity concerns. So, foundationalists can neither reject ESSENCE nor embrace circularity. The only remaining option is to reject BACKING.

3.3 *Rejecting the Backing Claim*

The third way to react to the circularity argument is to reject BACKING. If one denies that metaphysical explanations are backed by metaphysical determination relations, then there will be no requirement to include the relevant determination relation among the *explanans* in an explanation of a dependent entity's existence and nature. Thus, the circularity will be avoided.

6 I need to thank an anonymous referee for their helpful comments on the subject of circularity in relation to metaphysical infinitism.

Metaphysical holists, as stated above, can react to the circularity argument by embracing circularity. They are, therefore, under no pressure to deny BACKING. Metaphysical infinitists, however, have a choice to make in the face of the circularity argument. They can either hold onto BACKING, like metaphysical holists, or they can give up BACKING in order to retain an explanatory advantage over holism. Metaphysical infinitists, then, are under some pressure to deny BACKING in the face of the circularity argument. Metaphysical foundationalists cannot embrace circularity, so they are under substantial pressure to reject BACKING.

4 Conclusion

I argued that Cameron's argument for separating metaphysical explanations from metaphysical determination relations is not successful. I offered, instead, an alternative argument on the basis of premises Cameron makes use of, namely ESSENCE and BACKING. Cameron, in fact, comes very close to identifying the circularity in explanations of the existence and nature of dependent entities. He admits that <E1 ontologically depends on E2> is, by ESSENCE, among the things to be explained in the explanation for the existence and nature of E1 and continues to state that we can explain the existence and nature of E2 because E1 ontologically depends on E2 (Cameron 2022, 100). Yet he continues to formulate his argument on the basis of the idea that the success of each explanation is hostage to the next explanation, instead of dwelling on the circularity.

Besides reiterating the finitist dogma at the level of success of explanations, Cameron's argument, if it were sound, could motivate only metaphysical infinitists to separate explanations from determination relations. For, the success of explanation was threatened only in the infinitist case. The circularity argument, on the other hand, puts significant pressure on the metaphysical foundationalist to deny BACKING, along with putting some pressure on the metaphysical infinitist to do so. In conclusion, although Cameron's argument for separating explanations from determination relations fails, a better argument to do this job is available to him.*

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Grounding Ground and the (In-)Escapable Ill-Foundedness of the Inclusive 'Explains'

Yannic Kappes

The thesis that every grounding fact is grounded gives rise to an infinite series of grounding facts. According to Frugé ("Janus-Faced Grounding"), this series of grounds of ground amounts to a vicious regress. This paper (1) responds to Frugé's argument, (2) argues for a more plausible motivation for the vicious regress, and then (3) deploys a Bolzanian regress argument against this to defend the innocence of the series of grounds of ground.

Theories according to which every grounding fact is grounded give rise to the following kind of infinite series of grounding facts (let '$\prec$' express at least partial grounding):

$$Q$$
$$P \prec Q$$
$$\Gamma_1 \prec (P \prec Q)$$
$$\Gamma_2 \prec (\Gamma_1 \prec (P \prec Q))$$
$$\Gamma_3 \prec (\Gamma_2 \prec (\Gamma_1 \prec (P \prec Q)))$$
$$\dots$$

Here, let the Γ_i stand for whatever the grounds of the grounding fact in question are supposed to be: For example, according to Dasgupta (2014), these are certain essence facts; Sider (2020) holds that they can be of a more varied nature, while according to Bennett (2011), deRosset (2013b), and Litland (2017), they are the grounds involved in the grounding fact that is being grounded.[1]

1 In Litland's case, this results from *factive* grounding facts being grounded in the grounds involved together with the corresponding *non-factive* grounding facts, which are zero-grounded.

Against these accounts, Frugé (2023) attempts to show that the resulting infinite series of grounds of grounds constitute *vicious* regresses by arguing that they involve a kind of metaphysical dependence that allows to apply an analogue of Schaffer's (2010, 2016) consideration for the well-foundedness of grounding to the infinite series of grounds of ground. In what follows, I will develop Frugé's argument, argue against it, and draw a general lesson about the well-foundedness of metaphysical explanation from this discussion.

This is the plan: Section 1 presents Frugé's argument and argues that it fails. Section 2 discusses a related argument by (1) considerations akin to Frugé's counterfactual considerations, (2) introducing the inclusive sense of 'explains' and showing how each element in the series of grounds of ground is explained by its successor in this sense, and (3) arguing that the metaphor of explanation as a machine has a natural reading given which Schaffer's consideration for the well-foundedness of grounding applies *mutatis mutandis* to the infinite series of grounds of ground.[2] Against these considerations, section 3 fields a regress argument by Bolzano (2014b, para. 199) to argue for the innocence of the series of grounds of ground.[3]

1 Frugé's Argument

In a nutshell, Frugé (2023) argues for the viciousness of the series of grounds of ground by arguing (1) that what he calls a kind of "genuine dependence" holds between Q and all Γ_i in the series of grounding of ground (see above), and (2) that this allows applying (*mutatis mutandis*) Schaffer's consideration for the well-foundedness of grounding to reveal the viciousness of the series. I will now introduce the notion of well-foundedness and Schaffer's consideration for the well-foundedness of grounding, and then we will consider Frugé's argument in detail.

Proponents of the well-foundedness of grounding reject the existence of infinite regresses (i.e., downwardly non-terminating grounding chains) and circles of grounding, at least as long as the involved facts (or propositions, if you prefer) are not appropriately tethered to the fundamental. An important metaphorical consideration that motivates well-foundedness stems from

2 For the record: My aim is not to endorse Schaffer's consideration or the well-foundedness of grounding in this paper, but rather to defend the possibility of conjoining them with theories according to which every grounding fact is grounded.

3 In arguing against conceptions of grounds of ground like Dasgupta's (2014), Bennett (2017, 207) offers a related argument; see footnote 15 below.

Schaffer, according to whom grounding regresses are objectionable because in them, being would be "infinitely deferred [and] never achieved" (Schaffer 2010, 62):

> Grounding must be well-founded because a grounded entity inherits its reality from its grounds, and where there is inheritance there must be a source. [...] [S]omething cannot be real merely by having a limitless sequence of ancestors, each claiming reality from its parents. There must actually be a source of reality somewhere. (Schaffer 2016, 95)

I will call this 'Schaffer's consideration'.[4] Before we continue, note that infinite series of grounds of ground are not downwardly non-terminating grounding chains (and thus well-foundedness of grounding is not sufficient to argue against them): no element of these series is *grounded* in the next element. Accordingly, proponents of accounts according to which every grounding fact is grounded have insisted that the resulting infinite series are unproblematic and that we can accept their accounts while remaining neutral on whether grounding is well-founded.

Against this, Frugé (2023, sec. 2) argues that Schaffer's consideration *can* be extended to the infinite series of grounds of ground once we realize that Q metaphysically depends in a special way on each of the Γ_i (i.e., the grounds of the grounding facts in the infinite series of grounding grounds that starts with $P \prec Q$). Frugé calls this kind of dependence "connection dependence" and argues as follows:

> Why is connection dependence a genuine form of dependence? Suppose the following is the case: A grounds B, where C grounds that A grounds B. Then, B doesn't only depend on A. Instead, it also depends on C, because A only generates B given C. If there were no C to put A grounds B in place, then even if there were A there would be no B, since A wouldn't generate B because it wouldn't be the case that A grounds B. For example, if a collection of particles ground the composite whole of those particles only via a composition operation grounding this grounding fact, then if, perhaps counterpossibly, there were no composition operation

4 While I focus on Schaffer's consideration here, other arguments for the well-foundedness of grounding exist. An example of an argument meant to establish an aspect of well-foundedness is Kovacs' (2018) argument in favor of the irreflexivity of grounding.

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> then those particles would not ground that whole, because there would be no composition. (Frugé 2023, 976–977)

Having thus argued that Q metaphysically depends on Γ_1 in some genuine sense, he extends the argument to the rest of the Γ_i:

> Similar reasoning applies at each step in the stepwise path. If D grounds that C grounds A grounds B, then if there were no D, then even if there were C and A, then there would be no B because C would not generate that A grounds B, and so A would not generate B. And so on for each ground in the stepwise grounding path [i.e., in our terminology, the Γ_i]. Thus, connection dependence is a genuine form of dependence. B needs C in order to come about, and it also needs D in order to come about, and so on down the stepwise path. So B metaphysically requires each ground in its stepwise grounding path. (Frugé 2023, 977)[5]

Frugé then argues that (an analogue of) Schaffer's consideration applies:

> As Jonathan Schaffer says in the context of defending well-foundedness, if grounding did not terminate in an ungrounded ground, then "being would be infinitely deferred, never achieved" (2010: 62). But given connection dependence, then the same can be said for an infinite stepwise path of ever more grounding of grounding facts [i.e., series of ground of ground]. Even if ground were well-founded, if the grounding of grounding facts had no end, then 'being would be infinitely deferred, never achieved', since there would be no point at which it's ultimately settled that the grounded is generated. It would always need a further ground of a grounding fact. Therefore, if one thinks that violating the well-foundedness of ground is vicious, then one should also think that the fact regress is vicious—given that the grounded depends not just on its grounds but also on the grounds of its

5 Bennett (2017, 207) argues that whatever grounds P's grounding Q should also ground P. By considering grounding instead of "genuine metaphysical dependence," Frugé's counterfactual consideration can be understood as an argument for a generalization of this thesis. But as the discussion below suggests, there threatens to be an analogous consideration establishing that in addition to being grounded in P and the ground Γ_1 of $P \prec Q$, Q is also grounded in $P \prec Q$. While I am not sure what to think about Bennett's metaphorical consideration for her thesis, she rejects the latter result. See also footnote 15 below.

> grounding facts, and, more generally, on the grounds in its entire stepwise path of grounding facts. (Frugé 2023, 978)

Now, I believe Frugé's argument misses its mark: Even if we set all worries about his counterfactual argument aside and simply grant that Q depends in some genuine metaphysical sense on all of the Γ_i—in fact, we can even assume this relation to be grounding itself—it is hard to see how Schaffer's consideration *could* apply. For the structure of connection dependence that Frugé assumes is not even an infinitely descending chain down which "being could be infinitely deferred," but rather that of an infinite collection of Γ_i, on each of which Q depends, but which do not stand themselves in relations of metaphysical priority.

Since the structure of metaphysical priority that would seem to be required for Schaffer's consideration to apply is indeed that of an infinitely descending (and non-tethered) chain of dependence, one would have expected Frugé to argue that the elements of the series of ground of ground (i.e., those at the beginning of this paper) stand in a relation of metaphysical priority, but he does not do so. To an extent, this problem is perhaps obfuscated by Frugé's talk of "B metaphysically [requiring] each ground in its stepwise grounding path," which might suggest that this path consists in a chain of connection dependence holding between the elements of the series of grounds of ground (rather than the Γ_i), but this is not the case.

Additionally, Frugé seems to take issue with there seemingly being an infinity of Γ_i on which Q is connection dependent (e.g., "It would always need a further ground of a grounding fact."). But first, it is not easy to see what is supposed to be objectionable about this (many facts are only fully grounded in infinitely many facts taken together), and second, the assumption does not even follow, as the available accounts of the grounds of ground demonstrate: For example, on the Bennett-deRosset view, all Γ_i are identical to the original ground of Q, i.e., P in our case.

While I thus conclude that Frugé's argument fails, I will now show how one might attempt to substantiate in a different way the idea that series of ground of ground involve a relation of productive metaphysical priority that allows an analogue of Schaffer's consideration to apply and show the regress to be vicious.

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2 Three Better Considerations?

Let us discuss three considerations in favor of the thesis that the elements of the series of ground of ground stand in a relation of metaphysical priority (grounding or other) to which Schaffer's consideration applies. While I ultimately reject these considerations, I believe that they (or something close enough) are plausibly what motivates uneasiness about the series of grounds of ground.

2.1 *Counterfactual Considerations*

It may be tempting to think that a case can be made for the claim that each element of the series of grounds of ground counterfactually depends on its successor. For example, one might think that had $P \prec Q$ not been the case (and Q not been overdetermined by having a distinct further ground besides P), then Q would not have been the case. Moreover, one might even think that (counterpossibly) if P had been the case but $P \prec Q$ had not also been the case, then Q would still not have been the case. But while this is plausible for *some* instances, even setting overdetermination aside, counterexamples abound: For example, assuming $P \prec P \vee Q$, it is not in general the case that had $P \prec P \vee Q$ not been the case, then $P \vee Q$ would not have been the case: Even if $P \vee Q$ is not overdetermined (because only P is true but not Q), it might still be the case that if P had been false, then Q would have been true (and hence $P \vee Q$ too).[6] What is more, even if all such cases could somehow be excluded, counterfactuals do simply not map onto relations of metaphysical priority (at least not in the required way): For example, had $T(P)$ not been the case (let '$T(\,)$' be the truth operator), then P would not have been the case either; if anything, P has metaphysical priority over $T(P)$.

Now, rather than getting bogged down in thinking about counterfactuals further, let us consider two further attempts to argue that there is a relation of productive metaphysical priority (to which Schaffer's consideration applies) that holds between the elements of the series of grounds of ground—staying neutral for now on the question whether this alleged priority relation would be grounding or not; let us call it 'gog-priority'. The first attempt stems from an inclusive sense of 'explains' and the second from a particular reading of the metaphor of the machine.

6 Many thanks to the editors and an anonymous referee for discussion here.

2 The Inclusive Sense of 'Explains'

I assume that explanation why has the following tripartite structure (see, for example, Schaffer 2017):

> Base. A set of reasons why the explanandum obtains, e.g., causes or grounds.
>
> Link. An explanatory connection between the reasons in the base and the explanandum; these could either be instances of explanatory relations such as causation or grounding (call these 'type 1') or explanatory generalizations such as laws of nature or metaphysics, (explanatory) schemata, or (explanatory) inference rules (call these 'type 2').
>
> Explanandum. That which is being explained.

For an example, consider an explanation why a certain rose is red (Explanandum) in terms of its being scarlet (a ground that constitutes the explanation's Base) and the grounding fact of the rose's being scarlet grounding it's being red (or a metaphysical principle that states that instantiations of determinates ground instantiations of corresponding determinables).

In a restrictive sense, only the elements of the Base explain the Explanandum—the rose's being red is explained by its being scarlet, while the grounding claim or metaphysical principle plays a different (for example, explanation-backing) role. It is this restrictive sense that corresponds to 'because', which connects a sentence that expresses a reason why with a sentence that expresses an Explanandum (cf. Schnieder 2010; and Skow 2016).

But there also exists another sense of 'explains', in which links also (partially) explain their explananda. In this sense, the rose's being red is explained by it's being scarlet and the corresponding grounding fact of metaphysical principle together: Base and Link(s) together explain$_{\text{inclusive}}$ the Explanandum. This sense is, for example, operative in how the DN-model of explanation is often framed: Boundary conditions and laws (or lawlike generalizations) together form the explanans, and the explanans (or what is contained therein) explains the Explanandum (in the inclusive sense).

Equipped with this inclusive sense of 'explains' and assuming that instances of grounding correspond to instances of explanation, we can observe that the

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elements of the series of grounds of ground are (partially) explained$_{\text{inclusive}}$ by their successor, which is a type 1 LINK of a grounding explanation of its precursor. For example, Γ_1 is a ground of $P \prec Q$, and $\Gamma_1 \prec (P \prec Q)$ is a LINK of the corresponding grounding explanation. Therefore, $P \prec Q$ not only counterfactually depends on $\Gamma_1 \prec (P \prec Q)$ but is moreover partially explained$_{\text{inclusive}}$ by it.[7]

Hence, if there are series of grounds of ground, then the inclusive 'explains' allows for infinitely descending chains. Moreover, 'explains$_{\text{inclusive}}$' would presumably not be well-founded in the sense that any explained$_{\text{inclusive}}$ fact is ultimately explained$_{\text{inclusive}}$ by unexplained$_{\text{inclusive}}$ facts.[8] For consider a series of grounds of ground: Perhaps all elements could have a fundamental ground outside the series, but then the involved grounding relations give rise to further series (assuming that all instances of grounding are grounded, of course), and so on.[9]

Now, given the inclusive sense of 'explains' and the fact that the explanation in question is a metaphysical one, *some* metaphysical explanatory relation (i.e., a relation that can be called such in some good sense) holds between the elements of the series of grounds of ground. Together with the previous counterfactual observation, this could lead one to think that the relation in question is a relation of productive metaphysical priority, i.e., gog-priority. Assuming further that all such priority relations are subject to a variant of Schaffer's consideration, the viciousness of the series of grounds of ground would then follow.

3 *Metaphor of the Machine*

Indeed, I believe that there is something of a sense that Schaffer's consideration or a variant of it applies to gog-priority (if it applies at all) and that this can be brought out by a particular, yet arguably natural, way of construing the metaphor of grounding (or explanation) as a machine (cf. Litland 2017):

7 This consideration relies on there being type 1 LINKs rather than only LINKs of type 2; see the next subsection for discussion.

8 Even focusing exclusively on metaphysical explanation. The non-well-foundedness of explanation in general can arguably already be established on the basis of the non-well-foundedness of causation; cf. Schaffer (2016). This incidentally puts pressure on Frugé's (2023, 975) claim that "for any explanation both explainers and explanations must come to an end."

9 For a different kind of argument in favor of the thesis that the inclusive 'explains' (even restricted to metaphysical explanation) is not well-founded, see Hicks (2020) and Kappes (2022).

> Think of grounding (or explanation) as a machine: Instances of grounding are machines that take inputs (grounds) and use them to generate outputs (groundees). But for a machine to be able to generate something, it either has to exist without having been generated, or it has to be generated first. But this means that the series of grounds of ground corresponds to a series of machines, each generated by a previous machine, and so on *ad infinitum*. It seems like each machine inherits its reality from a further machine that generates it, and thus its reality is infinitely deferred and never achieved.[10]

Now, this understanding of the metaphor of the machine is not mandatory. First, it is just not clear why the causal-temporal relation between the machine and its output within the metaphor should have an analogue in a relation of productive metaphysical priority within reality: after all, metaphors break down somewhere, and this might well be where this one does.[11]

Second, as one of the anonymous referees for this paper has thankfully pointed out, if we assume explanatory LINKs only to be of type 2, that is, explanatory inference rules (as, e.g., Litland does) or certain laws (as, e.g., Schaffer does), rather than instances of grounding, we should presumably understand the metaphor as involving these rules or laws as their machines. But since, e.g., in Litland's case, roughly speaking, a general rule for grounding introduction is sufficient to generate all statements of higher order ground, no hierarchy of ever-descending explanatory machines is required. As the referee has pointed out, moreover, these accounts can avoid the regress of inclusive explanation: Roughly, in a case of P grounding Q, Q will be inclusively explained by P together with a metaphysical law or a statement concerning the validity of the relevant rule of explanatory inference (i.e., one linking P and Q), plus the law or statement concerning the validity of the rule that governs what grounds grounding facts. While this does not get rid of the corresponding infinite series of grounding facts, it does avoid any regress of inclusive explanation.

Now, while I find this very compelling, my aim in this section was to attempt to come up with possible reasons that could be what motivates uneasiness concerning the series of grounds of ground, and this I believe the above

10 Something like this *might* also underlie Frugé's (2023) temporal analogy.

11 Proponents of the well-foundedness of grounding who, like Schaffer, believe that causation is not well-founded have a further compelling reason for this.

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version of the metaphor of the machine achieves even in the light of the previous paragraph.

2.4 *Taking Stock*

The series of grounds of ground has been considered unproblematic by those committed to it (cf. Bennett 2011; deRosset 2013a; Dasgupta 2014; Litland 2017). For one, it is not an infinitely descending series of grounds and not obviously problematic in any other way. But more importantly, its proponents assume there to be a strong theoretical reason to allow for it: otherwise, it seems there must be at least some ungrounded grounding facts. But together with a principle of purity of the fundamental, this leads to the result that every entity (and other constituent of facts) is fundamental.[12]

Above, I have developed potential reasons in favor of the claim that the elements of the series of grounds of ground stand in a relation of gog-priority such that an application of Schaffer's consideration reveals the series to be objectionable. Though I ultimately reject these reasons, I take them to (1) provide a plausible diagnosis for the uneasiness concerning the series of grounds of ground that one occasionally encounters outside of print and which is likely shared by Frugé, and (2) substantiate this uneasiness to a point that is worth further engaging with.

While I have already mentioned some possible objections above, I will now argue that gog-priority runs into a version of Bolzano's regress.

3 With Bolzano against Gog-Priority

Let us suppose for the sake of argument that grounding is well-founded and that at least part of what reveals this is Schaffer's consideration. Given these assumptions, let us turn to gog-priority and see whether it is a notion to which an analogue of Schaffer's consideration applies (given the above considerations that motivated considering gog-priority as a genuine kind of metaphysical priority in the first place).

Now, either gog-priority just is grounding too or it is not: In the former case, insofar as Schaffer's consideration applies to grounding, so does it to gog-priority, because the latter just is (a subcase of) grounding. In that case, the

12 But see Correia (2023) and Barker (2023) for some challenges to purity, and Frugé (2023) for a non-trivial conception of fundamentality that allows for ungrounded grounding facts.

series of grounds of ground would constitute a vicious regress. Alternatively, gog-priority is a *sui generis* metaphysical priority relation besides grounding, for which we want to investigate whether Schaffer's consideration applies or not. Therefore, let us first consider whether gog-dependence could be grounding and then generalize our argument.

3.1 *Could Gog-Priority Be Grounding?*

I argue that in this case, grounding facts are not fully grounded in their conventional grounds (the non-gog-priority grounds), and at least the corresponding higher-order grounding facts (which are gog-priority grounds) must be added. For example, $P \prec Q$ will not be fully grounded in its conventional grounds Γ_1; rather, it would at least require $\Gamma_1 \prec (P \prec Q)$ too. This can be brought out by reflection on the metaphor of the machine as construed in the previous section (relying on this construal of the metaphor seems dialectically appropriate since I have diagnosed it as underlying the idea that gog-priority exists as a genuine relation of metaphysical priority subject to Schaffer's consideration).

On this understanding, both the input of the machine and the machine (i.e., the grounding fact) ground the output. In a way, the causal relations that hold within the fiction of the metaphor between the input and the output as well as the machine and the output stand, on this view, simply for grounding. But again, within the metaphor, input and machine can cause the output only together. Therefore, on this understanding, it would seem that the metaphor suggests that Γ_1 and $\Gamma_1 \prec (P \prec Q)$ somehow ground $P \prec Q$ together, neither on its own sufficient as a full ground. Additionally, if we assume otherwise, it would seem that applying Schaffer's consideration would not get us the right result: something that has a full fundamental ground (as $P \prec Q$ would have via its ordinary ground Γ_1) surely has "achieved being"; there being an infinitely descending chain of further grounds would not seem to detract from this.[13]

Now, to simplify, we write 'P' for 'Γ_1' and 'Q' for '$P \prec Q$' (but also consider that if the full grounds of all grounding facts must contain something like $\Gamma_1 \prec (P \prec Q)$, then it seems plausible that something analogous holds for all cases of grounding). On the first level, we thus have (let '$\prec$' express at least

13 Granted, lest a further problem of reality achieving happen somewhere along that infinitely descending chain, all of its elements must be fully grounded in something fundamental.

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partial ground):

$$P, (P \prec Q) \prec Q$$

Here is the crux: If P can only ground Q together with help from $P \prec Q$, it would also seem that $P, (P \prec Q)$ cannot fully ground Q alone! Rather, it seems that they too would need help, namely, from $P, (P \prec Q) \prec Q$. At least, the alternative seems objectionably ad hoc: If $P \prec Q$ is indeed a ground, how come $P, (P \prec Q)$ can fully ground Q, while P cannot?

This result (i.e., that $P, (P \prec Q)$ cannot be a full ground of Q because it does not contain a LINK-like element that takes us from $P, (P \prec Q)$ to Q) can be supported by at least one of the considerations that originally motivated that there is something problematic about the series of grounds of ground: Within the metaphor of the machine construed as above, input and machine together *cause* the output. According to my diagnosis, this is what underlies the idea that the relation of gog-priority (which we have here identified with grounding) holds between $P \prec Q$ and Q. But then it seems that we should be able to apply the metaphor to $P, (P \prec Q) \prec Q$ too: This instance of grounding corresponds to a machine that takes P and the original machine (corresponding to $P \prec Q$) as inputs and gives out Q. But then the current understanding of the metaphor delivers that Q is also at least partially grounded in $P, (P \prec Q) \prec Q$, we can apply the metaphor again, and so on!

Now, if what I have just said is correct, then we run into a version of Bolzano's (2014a, para. 199; cf. Rusnock and George 2014) regress:

$$P \prec Q$$

$$P, (P \prec Q) \prec Q$$

$$P, (P \prec Q), (P, (P \prec Q) \prec Q) \prec Q$$

$$\ldots$$

Bolzano outright rejects (his version of) this series as incoherent, but he does not provide an argument. I submit that there is at least some intuitive strangeness to this series, and while this might not be a particularly strong reason in general, it may have more bite in the present context where we argued against a position itself to a good part motivated by similar intuitions.[14]

14 Perhaps it could be possible to tell a story that actually supports the innocuousness of Bolzano's series by developing the idea that the new partial ground that is added at every step in the series somehow builds upon the previous partial grounds, thus getting us closer and closer to Q and reaching it at the limit? Thus understood, P gets us to some extent to Q, its getting us there to some extent gets us a little further, and so forth, until at the limit, we reach Q.

Now, for our opponent, who set out to avoid an allegedly problematic infinite series, the situation is already somewhat awkward, but could they perhaps bite the bullet and declare the plurality of all the grounds constructed above to be a full ground Ω of Q? I do not think so because, presumably, we should apply the crux argument to Ω too. But by doing so, we seem to reveal that Ω cannot be a full ground of Q either: in Ω, there is no grounding fact that takes us from all the grounds in Ω to Q! Yet, as I have argued above, this is what the above construal of the metaphor of the machine would require: Within the metaphor, the machine that takes Ω as input and gives out Q—it causes Q together with the Ω. But since causation is the metaphorical analogue of grounding here, the grounding fact that takes us from the Ω to Q would have to be included in a full ground of Q—yet, it is not among the Ω!

One might now consider whether a full ground of Q could be obtained from Ω by some transfinite construction similar to how Ω was constructed, but as long as the result is such that we can say something that amounts to those grounds (i.e., those resulting from the construction) grounding Q, it looks like we can apply the crux and obtain a further grounding fact that should be part of the full ground but was not constructed. Thus, unless declaring full grounds to be ineffable and giving up talking about them like above is considered an option, I conclude that Bolzano's regress must be avoided.

In this subsection, I have argued that given (1) the motivation (from the previous section) for gog-priority being a genuine kind of metaphysical priority to which Schaffer's consideration applies and (2) the assumption that gog-priority just is grounding, Bolzano's regress arises. Since Bolzano's regress must be avoided, (1) is undermined given (2). Next, we will drop assumption (2) and argue that a Bolzanian regress arises (and hence (1) is undermined even if gog-priority is not grounding).[15]

15 Mentioning Carroll's related regress, Bennett (2017, 207) offers a similar argument against theories like Dasgupta's, according to which grounding facts are grounded in principles that connect grounds with groundee. Her argument relies on the claim that whatever grounds a grounding fact $\Gamma \prec Q$ must also ground Q (for a (pre-emptive) response to Bennett's regress, see Dasgupta 2014, 587–589). While I cannot assess Bennett's metaphorical consideration for that claim here, a likely upshot of our present discussion is that it must not generalize to the grounding facts themselves: Claiming that $\Gamma \prec Q$ must be a ground of Q is what gets our regress argument going.

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3.2 *Suppose Gog-Priority Is* Not *Grounding*

To round off the argument, assume now that gog-priority is *not* grounding but some *sui generis* kind of metaphysical priority relation. Suppose Q is fully grounded in a fundamental fact P and gog-posterior to $P < Q$ (i.e., P's fully grounding Q is gog-prior to Q). Let us consider how Schaffer's consideration might apply in this situation: Suppose first that something can "achieve reality" already by being fully grounded in something fundamental. Then that thing's *additionally* being located at the top of an infinitely descending, non-terminating chain of gog-priority would not seem to impact Q's being real—Q would have already "achieved reality" by being fully grounded in something fundamental.

Therefore, it seems our opponent should rather hold that neither only something's being *grounded* in something fundamental nor only its being gog-posterior to something fundamental can be sufficient for the thing's having achieved reality, or (we might say) "having been made real." In our example case, this means that while P fully grounds Q and Q is gog-posterior to $P < Q$, neither P nor $P < Q$ on its own is sufficient to make Q real. Rather, P and $P < Q$ only make Q real together.

Now, whatever this relation of real-making would be (perhaps it could just be the disjunction of grounding and gog-priority?), it looks like the opponent of the series of grounds of ground must hold that an analogue of Schaffer's consideration applies to *it*, and hence that for something to be real, it must either not be made real by anything or be fully made real by some things that are not made real by anything.

But then it seems like we can run the crux argument with real-making instead of grounding and thus construct the Bolzanian regress for this relation of real-making: If P can only make Q real together with help from $P < Q$, why believe that P, $(P < Q)$ can fully make Q real on their own? It would seem that they also need help, namely, from some fact concerning P and $P < Q$'s (partially) making Q real. At least, the alternative seems ad hoc: If $P < Q$ indeed (partially) makes Q real, how come P and $(P < Q)$ together can fully make Q real, while P alone cannot?

It seems that the reasons our opponent has to believe that P can make Q real only together with $P < Q$ seem to carry over to P's and $P < Q$'s together making Q real: Consider once the metaphor of the machine as construed above, within which input and machine together *cause* the output. This causal

relation is the metaphorical equivalent of the relation of real-making that we are currently considering.

But then our opponent would have to produce a good reason to stop the metaphor thus understood from applying to real-making as well: Like grounding, real-making can metaphorically appear as a machine that takes inputs (the "real-makers") and puts out real things. But why would the metaphorical parallel between causation and grounding in the original (stretched) metaphor then not have its equivalent here? If our opponent cannot produce a good answer, it would seem that P's and $P < Q$'s making Q real itself would have to be a real-maker of Q, and by similar reasoning to what we used above in the case of grounding, it would then seem that P and $P < Q$ alone cannot fully make Q real. Rather, they would at least have to be accompanied by *their* making Q real, thereby starting the Bolzanian regress for real-making.

Again, one might criticize this argument for its dependence on an intuitive assessment of a particular understanding of the metaphor of the machine. But consider the dialectical situation once more: This is the very kind of consideration that, according to my diagnosis in section 2, underlies the idea that the series of grounds of ground is problematic. Thus, at least unless the opponent of the series of grounds of ground comes up with a different argument, they are confronted with the Bolzanian regress whether they understand gog-priority as grounding or as a *sui generis* relation of metaphysical priority.

4 Conclusion

Let us recapitulate: Section 1 argued against Frugé's argument for the viciousness of the series of ground of ground: Even if his notion of connection dependence corresponds to a genuine form of metaphysical priority, the resulting structure does not allow for an application of Schaffer's consideration to the series of ground of ground.

Section 2 discussed three related considerations in favor of the thesis that a well-founded relation of metaphysical priority holds between the elements of the series of ground of ground. These concerned certain counterfactuals, the inclusive sense of 'explains', and a natural reading of the metaphor of explanation as a machine. While I ultimately argued that we should not endorse these three considerations, I suggested that we should take them seriously as likely underlying the claim that the series of ground of ground is vicious.

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Finally, in section 3 I argued that endorsing the viciousness of the series of ground of ground based on the considerations identified in the previous section runs into a variant of Bolzano's regress. Hence, we should reject the problematic considerations and can maintain the innocence of the series of ground of ground.*

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