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Criteria of Identity

Ground, Essence, Abstraction

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A criterion of identity for *F*s states that two objects of *F*s are identical if and only if those very objects stand in a certain relation R_F. This paper proposes an essentialist account of what it is to be a criterion of identity: for a statement to be a criterion of identity for *F*s it has to be essential to being an *F*, as such, that two *F*s are identical iff they stand in R_F. We argue that this account is superior to recently proposed ground-theoretic accounts. We then apply this account to throw new light on abstraction principles.

A criterion of identity states that two objects of sort F are identical if and only if those very objects stand in a certain relation R_F. Some have suggested that such criteria should be understood epistemically (see e.g., **?**), others semantically (see e.g., **?**), and others metasemantically (see e.g., **?**; **?**). This note concerns metaphysical interpretations of criteria of identity. Recently, several authors (see e.g., **?**; **?**; **?**; **?**; **?**) have proposed that criteria of identity can be understood as metaphysical principles about the *grounds* of identity facts: it is *because* $R_F(x, x)$ that $x = x$.

Negatively, we argue against ground-theoretic construals of criteria of identity (section 1). Positively, we propose an alternative essentialist account and shows that it avoids the problems of the ground-theoretic accounts (section 2). We then (section 3) apply this essentialist account to throw new light on abstraction principles; in particular, we argue that they must be formulated without using functional expressions. Section 4 concludes.

1 Against Ground-Theoretic Criteria

A one-level identity criterion has the following form:[1] Such a criterion thus says that two objects of sort F are identical if and only if those very objects stand in a certain relation R_F. The relation R_F will differ from sort to sort: for *fusions* the relation might be having the same parts; for *persons* (at a time) it might be (non-branching) psychological continuity, for *sets* it will be the relation of having the same members, and so on.

It will be useful to begin by by considering *minimalist* accounts of criteria of identity. A *minimalist*—see e.g., (**?**)—takes every necessarily true instance of to be a criterion of identity. However, as observed by (**?**) and (**?**), this allows criteria of identity that are too strong. For suppose there are at least two Fs and that

$$Fx \wedge Fy \rightarrow (x = y \leftrightarrow R_F(x, y))$$

is a criterion of identity for Fs. Let ϕ be any necessary truth. Then

$$Fx \wedge Fy \rightarrow (x = y \leftrightarrow (\lambda zu.(R_F(z, u) \vee \neg\phi))xy)$$

is also a necessary truth of the form . But this "criterion" of identity for Fs together with the fact that there are two Fs entails ϕ! That means that for each necessary proposition ϕ there is a criterion of identity for Fs that entails ϕ. But even if there are many correct criteria of identity for persons one would have thought that no criterion of identity for *persons* should have as a logical consequence that there are inaccessible cardinals.

One motivation for the simple ground-theoretic view of criteria of identity is to avoid this problem. This account requires that the (instances of the) right-hand-side of the embedded biconditional in *ground* the (instances of the) left-hand-side. [2] For instance, if x, y are two sets it is *because* they have the same members that they are identical. Since ground is an explanatory notion such a view avoids the problem with inaccessible cardinals: clearly,

1 See (**?**) for the distinction between one- and two-level criteria.

2 We have to make some decisions about how to express claims of ground. Officially, we will take ground to be expressed using a sentential operator (**?**; **?**), which we may pronounce "because". For ease of expression we will however freely nominalize and talk about ground as a relation between *facts*. When we say that the fact that $a = a$ is grounded in the fact that q this should be understood as saying that $a = a$ because q. Quantification over facts should be understood in higher order terms. When we say "every fact is such that …" this should be understood as saying "$\forall p(p \rightarrow \dots)$"; and similarly for existential quantification over facts.

it is not partly *because* there are inaccessibles that two distinct persons are distinct!

Unfortunately, this ground-theoretic account conflicts with a number of plausible views about the grounds of identity facts. These views are all (partly) motivated by the following common intuition.

Identity is a very simple relation: it holds between each object and itself and between no other objects (**?**). The true identity-propositions are exactly the ones of the form $a = a$. (True propositions of the form $a = b$ are, by Leibniz's Law, identical to propositions of the form $a = a$.) Consider some object a. It does not matter what sort of object a is—it will be a fact that $a = a$. Why should the fact that a is of kind F make a difference to how $a = a$ is grounded? Or take two different objects a, b. It does not matter what kinds of objects they are: it will still be the case that $a \neq b$. Why should the fact that $\neg R_F(a, b)$ make a difference to how $a \neq b$ is grounded?

These considerations motivate the search for a *uniform* account of the grounds for identity facts, an account on which all identity (distinctness) facts are grounded in the same way. Recently, several such uniform accounts have been developed.[3]

(**?**) has proposed that identity facts $a = a$ are (uniquely) zero-grounded (and that distinctness facts $a \neq b$ are (uniquely) zero-grounded). (**?**) has proposed that $a = a$ is (uniquely) grounded in the *existence* of a.[4] (**?**)—adopting the heterodox view that objects can ground facts—argues that $a = a$ is grounded simply in a. Finally, though he does not commit himself to this explicitly, it is natural to take (**?**; **?**) to hold that identity facts are "autonomous": while they are ungrounded, they have the special status of not even being "apt to be grounded". (Indeed, if one believes in autonomy identity propositions would appear to be one of the best candidates for having this exalted status.)

3 Another uniform view would take all identity and distinctness facts to be ungrounded. While the essentialist account in this paper is compatible with this view, we are inclined to reject because of Purity considerations. The Purity Principle (**?**) states that non-fundamental objects do not figure in ungrounded facts. If identity-facts were ungrounded every object would then be fundamental. Both Dasgupta's, Litland's, and Rubeinstein's accounts allow us to retain the Purity Principle while allowing there to be non-fundamental objects. For more on desirability of having uniform accounts of the grounds of identities, see (**?**; **?**). For an overview of the debate about the grounds of identity facts and further discussion of the options, see (**?**; **?**).

4 This idea was first suggested by (**?**).

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All these views are incompatible with the simple ground-theoretic account of criteria of identity.[5]

What about the more sophisticated ground-theoretic construal of criteria of identity developed by (**?**)? Fine thinks that criteria of identity should be formulated using a *generic* notion of ground. For him, the task is not to specify the grounds for particular identities like $a = a$, where a may be an F; our task is rather to say what makes two arbitrary Fs the same. To illustrate: we are not primarily interested in grounding a fact like $\{1, 2\} = \{1, 2\}$ in the fact every member of $\{1, 2\}$ is a member of $\{1, 2\}$. We are rather interested in knowing what makes two arbitrary sets identical.

We do not have an objection to the notion of generic ground as such;[6] however, generic ground does not help make sense of criteria of identity. The problem is that generic ground has to be connected to particular ground. As (**?**) notes the natural idea is that instances of the generic grounding claim are themselves particular grounding claims.[7] But then the fact that $\{1, 2\} = \{1, 2\}$ ends up being grounded in the fact that every member of $\{1, 2\}$ is a member of $\{1, 2\}$. This commits us to the non-uniform view about the grounds of particular identity facts found problematic above.

While these considerations are not decisive, they provide us ample reason to look for an alternative metaphysical account of criteria of identity.

2 An Essentialist Account

Our alternative is essentialist: for a statement of the form to be a criterion of identity for Fs it has to be essential to being an F, as such, that two Fs are identical iff they stand in R_F.[8] The idea that criteria of identity should be understood in essentialist terms has been suggested before—see e.g., (**?**)—but

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5 Another view which is incompatible with the simple ground theoretic account is the Leibnizian view that the grounds for $a = b$ is the indiscernibilty fact that a has all the same properties as b. The standard objection against the Leibnizian view is that it circular: one of the properties a has is the property $\lambda x.x = a$ of being identical with a. Standard views in the logic of then gives rise to a circle of ground: $a = a$ is grounded in $(\lambda x.x = a)a$ which in turn is grounded in $a = a$. Recently, (**?**) has defended a version of the Leibnizian view by invoking the notion of "proxy-grounding". (This notion was introduced by (**?**).) A fuller discussion of Elgin's view has to await another occasion.

6 For applications of notions like generic ground see (**?**) and (**?**).

7 Fine notes that some restrictions have to be put on the instances, but the restrictions he has in mind does not help with the current issue.

8 This essentialist locution from is adopted from (**?**).

the view has not yet been rigorously developed. To do so we adopt the language of the (higher-order) logic of essence.[9] When F is a property, we write $\Box_F$ for the essentialist operator "it is true in virtue of the nature of being F that ..." Two clarifications about the relevant notion of essence are in order.

First, since we want to give a metaphysical interpretation of criteria of identity it is important that the notion of essence be *worldly*: if $\Box_F \phi$ and the proposition expressed by ϕ is the same as the proposition expressed by ψ then $\Box_F \psi$; in fact, the proposition expressed by $\Box_F \phi$ should be identical to the proposition expressed by $\Box_F \psi$.[10]

Second, we are working with a notion of *mediate* essence. To explain this, we must introduce the notion of ontological dependence. Say that *a rigidly essentially depends* on F iff F figures in a proposition true in virtue of the nature of a (**?**).[11] (Formally, a rigidly depends on F iff $\exists P \Box_a P(F)$.) We will write $a \succeq F$ for the claim that a depends on F.[12] Our notion of essence is mediate in the sense that if $a \succeq F$ then any proposition which is true in virtue of the nature of F is also true in virtue of the nature of a. This is (a special case of) the principle of Chaining. [13]

We thus propose that a criterion of identity for Fs has to satisfy the following essentialist constraint: Like the ground-theoretic account this avoids the problem of inaccesible cardinals: clearly, it is not part of what it is to be a person, in general, that two persons are identical if they are psychologically continuous or there are no inaccessible cardinals. Above we objected to minimalism on the ground that it permits criteria of identity that are too *strong*—in the sense of entailing that there are inaccessibles. From the essentialist point of view, this should be seen as a symptom of the deeper problem that a criterion of *personal* identity should say nothing about cardinal numbers; it should not even make the trivial claim that either there are or there are not inaccessibles.

A natural proposal is then: a statement of the form is a criterion of identity for Fs iff it satisfies . However, this is not satisfactory for two reasons.

9 See (**?**) for a precise statement of this language and a characterization of the logic.

10 That we are able to apply Leibniz's Law in essentialist contexts in this way will be important in section 3 below.

11 Since we are working in a higher-order logic of essence F and a can be of any types.

12 Strictly speaking, we have a typed family of different relations. For any types τ, σ we have a dependence relation $\succeq^{\langle \tau, \sigma \rangle}$ that holds between entities of type τ and entities of type σ.

13 While the Chaining Principle is more general, this special case suffices for present purposes. For a fully general formulation of Chaining the reader is referred to (**?**).

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First, while we disagree with Fine that criteria of identity involve claims about generic *ground*, we are inclined to agree that they are *generic*. This generic character is not captured simply by meeting . To see this consider:

meets . However, simply makes many claims about individual sets: for *any two sets* x, y it claims that they are identical iff they have the same members. But we want a criterion of identity to make a claim about sets *as such* and not many claims about individual sets. This suggest that a criterion of identity should not simply meet an essentialist constraint but should itself be essentialist *in content*. The criterion of identity for sets should thus be the relevant instance of . Thus:[14]

Second, we should not simply take a criterion of identity to be a true instance of . Let H be the property of being human and R_H be the relation that figures in the identity criterion for humans. Consider the property H^* of being human while there are inaccessible cardinals or not ($\lambda x.Hx \land (\exists yIy \lor \neg\exists yIy)$). It will be essential to being H^* that any two H^*s are identical if and only if they stand in relation R_H. However, one might think that

$$\Box_{H^*} \forall x \forall y (H^* x \land H^* y \rightarrow (x = y \leftrightarrow R_H(x, y)))$$

is not a genuine criterion of identity.

Say that a property F is a *sort* if it lies in the nature of each F that is is an F.[15] H^* is not a sort in this sense: the H^*s are exactly the humans, but it is not part of the nature of each human that either there are or there are not inaccessible cardinals.[16]This suggests that we should require not just that the relevant instance of be true but also that the following principle is true:[17]

14 While the embedded generalization in is not generic we take it that embedding this generalization under the essentialist operator results a generic claim. If one is not satisfied with this way of expressing generic claims there are alternatives. (**?**; **?**) suggests three ways of expressing generic essentialist (and grounding) claims:

using arbitrary objects;

using conditional essence claims; and

using restricted λ-abstraction.

Moreover, (**?**) sketches an account of generic definitions using a variable-binding definitional operator. The claims made in this paper could be reformulated in either of these ways. However, unlike the higher-order logic of essence, neither of these approaches are worked out in full formal detail. We have therefore elected to work with the logic of essence.

15 *Sorts* are the worldly analogue of what (**?**) call a "pure sortal concept".

16 It is here crucial that we are working with a notion of essence, that is not closed under logical consequence. See (**?**) for a discussion of the distinction between constitutive and consequential essential.

17 Perhaps one should require more than that F is sort. It is, e.g., somewhat intuitive to think that the criterion of identity for *mammal* should be the same as that for *animal*. If so one might want

While criteria of identity are primarily generic claims about what, in general, it is to be an F, they do have consequences for particular Fs.

From and we can derive the following claim about identity-propositions involving particular objects:

$$\forall x \forall y (Fx \wedge Fy \rightarrow \Box_{x,y}(x = y \leftrightarrow R_F(x, y)))$$

To see that this follows we reason as follows.[18] Let a, b be arbitrary such that Fa and Fb. By we have $\Box_a Fa$ and $\Box_b Fb$. Thus a, b both depend on F. By the Chaining Principle noted above anything true in virtue of the nature F is true in virtue of the nature of what depends on F. We thus get that $\Box_{a,b} \forall x \forall y (Fx \wedge Fy \rightarrow (x = y \leftrightarrow R_F(x, y)))$. While the essence of some items $a, b, c, \ldots$ is not closed under logical consequence, it is closed under the logical consequences that hold in virtue of the items occurring in the essence of the items in question.[19] We thus get that $\Box_{a,b}(a = b \leftrightarrow R_F(a, b)))$. We thus have proved $Fa \wedge Fb \rightarrow \Box_{a,b}(a = b \leftrightarrow R_F(a, b))$. Since a, b were arbitrary, we can conclude $\forall x \forall y (Fx \wedge Fy \rightarrow \Box_{x,y}(x = y \leftrightarrow R_F(x, y)))$.

Since we have $\forall x (Fx \rightarrow \Box_x x = x)$[20] it also follows that:

$$\forall x (Fx \longrightarrow \Box_x R_F(x, x))$$

One might think that this is a trivial claim—after all, R_F is, presumably, necessarily an equivalence relation. However, while $\forall z R(z, z)$ is a necessary truth, this truth need not be part of the essence of x.

We can also derive the following corresponding claim about distinctness:

$$\forall x \forall y (Fx \wedge Fy \wedge x \neq y \rightarrow \Box_{x,y} \neg R_F(x, y))$$

This claim, too, is non-trivial.

to insist that F is a category in the sense of (**?**). Imposing this stronger requirement will not affect the points made in this paper.

18 This informal reasoning can be carried out in Ditter's higher-order logic of essence.

19 This is the restricted closure principle RC from (**?**) which ensures that if ψ is a logical consequence of ϕ then $\Box_{a,b,c,\ldots}\psi$ is a consequence of $\Box_{a,b,c,\ldots}\phi$ if all the constants and free variables in ψ are also in ϕ.

20 It is a theorem of the logic of essence that $\Box_{x,=} x = x$. That is, it lies in the nature of x together with the identity relation that $x = x$. However, by F depends on $=$ so it follows by Chaining and that if Fx then $\Box_x x = x$.

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3 Two-Level Criteria and the Nature of Abstraction Operations

Much of the recent interest in criteria of identity have concerned not one-level criteria like , but rather so-called two-level criteria. These are standardly taken to have the following functional form: A two-level criterion thus says that the values of a function f on some inputs are identical if the inputs stand in a certain relation R_f. The great interest in two-level criteria stems from the large literature on abstraction principles that has sprung up in the wake of the neo-fregean program in the philosophy of mathematics inaugurated by (**?**). (Formally, abstraction principles are just two-level criteria of identity.)

The relation R_f depends on the function f: if f is the function of direction abstraction then R_f would be the relation of parallellism; if f is the function of cardinal abstraction, then R_f would be the equinumerosity relation; and if f is the function of set-formation, R_f would be the relation of coextensionality.

We will now argue that for the purposes of metaphysics functional formulations of the criteria of identity are unacceptable.[21] Suppose that sets are formed by applying the set-formation function SetOf to (small) pluralities of objects.[22] Thus SetOf(xx) is the set that contains all and only the objects amongst the xx as members.[23]

We then have the following functional two-level identity-criterion for sets:

Consider now the singleton of Socrates, or {Socrates}. It is natural to hold that it is essential to {Socrates} that it is formed by applying SetOf to Socrates. Put in functional terms:

But how should the term "SetOf(Socrates)", as it occurs in , be interpreted? If this term just stands for its value—that is: {Socrates}—an application of Leibniz's Law ensures that the proposition expressed by is identical to: The proposition expressed by is, of course, true. The problem is that since $\forall x \Box_x x = x$ is a theorem of the logic of essence we have that for all objects a the identity proposition $a = a$ is part of essence of a. The proposition expressed by thus does not say anything interesting about the nature of {Socrates}, in particular.[24]

21 A different, more general, argument against the use of functions in metaphysics is given by (**?**). Of course, for many applications it is technically convenient to have functional *expressions*—but functional expressions can be treated as definite descriptions in the manner of Russell.

22 For an early statement of this view see (**?**).

23 In case one does not like this metaphysics of sets, the point can be made using other abstraction operations.

24 We should note that, strictly speaking, $\forall x \Box_x x = x$ is not a theorem of the *higher-order* logic of essence, only the weaker $\forall x \Box_{=,x} x = x$ is. Does this make a difference? Not really.

So perhaps “SetOf(Socrates)” does not stand for its value, but rather stands for a “complex” that has both the set formation operation SetOf as well Socrates as constituents?[25]

The problem with this view is that it would render “SetOf(Socrates)) = {Socrates}” false! For on the “right” we have the value of the function and on the left we have a certain complex. To save the view one thus has to say that in “SetOf(Socrates) = {Socrates}” the identity-symbol does not stand for the identity relation but rather stands for a relation like *has the same value as*. But then the identity relation does not figure in the criterion of identity![26]

Of course, this argument against functional identity criteria relies on being able to use Leibniz’s Law in essentialist contexts; one could thus block the argument by treating essentialist contexts as opaque. If essence is to be a worldly phenomenon, Leibniz’s Law is, however, non-negotiable: if essentialist contexts are opaque it does not make sense to ask what the essence of an object is independently of a particular presentation of the object. If the functional formulation was the only way of expressing two-level identity-criteria, perhaps one would have to live with an opaque notion of essence, However, two-level identity-criteria we can be adequately formulated in a relational manner.

To take the example of sets, let us write SetOf(xx, y) to mean that y is the set formed from the xx. The essentialist claim about {Socrates} can then made as follows: More generally, a relational two-level criterion of identity governing a relation F will say that it is essential to F that for any x, y, z, u if $F(x, z)$ and $F(y, u)$ then $z = u$ iff $R_F(x, y)$. We thus have the following generalization of :

We also have to generalize . Suppose $F(x, y)$ then it is essential to y that there is z such that $F(z, y)$. Moreover, that this is so is part of the essence of F. Thus:

$\Box_{\{\text{Socrates}\}}$\{Socrates\} = \{Socrates\} is simply an instance of the claim that every object that depends on the identity relation is essentially self-identical.

25 This idea is adopted by (**?**) and (**?**). See also (**?**).

26 Strictly speaking, one only requires that in (2) the identity symbol “=” does not stand for identity. One could hold that “=” stands for identity in “SetOf(Socrates) = {Socrates}”. The cost of doing so is to hold that “SetOf(Socrates)” is ambiguous. In non-embedded contexts it stands for its value; embedded under the essentialist operators $\Box_F$ it rather stands for a complex. Clearly, positing such an ambiguity in “=” and “SetOf(Socrates)” should be a last resort.

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Once we have formulated two-level criteria relationally, they can also be expressed as one-level criteria.[27] Take a two-level criterion

$$\Box_F \forall x \forall y \forall z \forall u (F(x,z) \wedge F(y,u) \rightarrow (u = z \leftrightarrow R_F(x,y)))$$

Let G be the property $\lambda u.\exists y F(y,u)$. And let R_G be the relation $\lambda zu.(\exists xy(F(x,z) \wedge F(y,u) \wedge R_F(x,y))$. The two level criterion then gives rise to the one-level criterion

$$\Box_G \forall u \forall z (Gu \wedge Gz \rightarrow (u = z \leftrightarrow R_G(u,z)))$$

We now want to make some observations about the essence of different abstraction operations. We will only be interested in abstraction operations that are *generative* in the sense that a value of the operation "is the result of applying the operation, not just in the innocuous sense of being identical to the result, but also in the philosophically significant sense of having its identity thereby explained." (**?**). Suppose Σ is such an abstraction operation and $\Sigma(x,y)$. This is no accident; rather, there is way that x is, and whenever some z is that way we have $\Sigma(z,y)$. More precisely, we propose that if $\Sigma(x,y)$ then there is a property P such that x has P and P is independent of y (though not necessarily of x) and it lies in the nature of y together with the property P that y can be abstracted from all and only the z that have P. Formally:[28]

Why do we require—supposing that $\Sigma(x,y)$—that the property P be independent from y? This is required by the generativity of Σ. We can explain what y is by saying that y is value of Σ applied to any z with property P. But this explanation would be circular if the property P depended on y.

As we shall see is the strongest claim that holds in general, though for some abstraction operations stronger claims hold. It will be instructive to look at two representative cases.

Consider first the case of sets and suppose that xx, y are such that $\mathrm{SetOf}(xx,y)$. Since a set can be formed only from its members the property P required by is the property of being the xx—$\lambda yy.yy = xx$. We thus obtain: $\Box_{\lambda yy.yy=xx,y} \forall zz(zz = xx \leftrightarrow \mathrm{SetOf}(zz,y))$.

Since it is essential to a given set that it is formed from the objects from which it is in fact formed, it is also essential to y that it is the result of applying SetOf to any zz with $\lambda yy.yy = xx$. Thus y depends on $\lambda yy.yy = xx$ and so, by Chaining, we get the following stronger claim:

$$\Box_y \forall zz(zz = xx \leftrightarrow \mathrm{SetOf}(zz,y))$$

27 We here agree with (**?**) and (**?**) though the reduction of two-level to one-level criteria is different.

28 Why in the nature of y, P *together*? See below.

Moreover, this reasoning does not turn on xx, and y in particular; thus we obtain the following general claim about the nature of set-formation itself:

$$\Box_{\text{SetOf}}\forall xx\forall y(\text{SetOf}(xx,y)\to\Box_y\forall zz(zz=xx\leftrightarrow\text{SetOf}(zz,y)))$$

Consider next the case of cardinal abstraction. The cardinal number 2 can be abstracted from the plurality of Biden and Trump; it can also be abstracted from the plurality of Nixon and Humphrey. It is, however, not plausible that it is essential to 2 that it can be abstracted from the plurality of Trump and Biden. (The number 2 "knows nothing" about particular Americans.) This difference between cardinal abstraction and set formation is usefully put in terms of ontological dependence: while a set depends on its members a cardinal number does not depend on the pluralities from which it can be abstracted.

The property P in is the property of being a plurality with exactly two members. It lies in the nature of 2 that it can be abstracted from any plurality with that property. This property is definable in purely logical terms and so does not ontologically depend on the number 2. There is nothing special about the number 2 here. For any number n, it lies in its nature that it can be abstracted from all and only pluralities that have property of having exactly n members, where this property does not depend on the number n. Writing # for the operation of cardinal abstraction we thus have

$$\Box_{\#}\forall xx\forall y(\#(xx,y)\to\exists P(P(xx)\wedge P\not\succeq y\wedge\Box_y\forall zz(P(zz)\leftrightarrow\#(zz,y))))$$

In the cases of set formation and cardinality abstraction the following stronger principles also hold:

It is important so see that these principles do not hold in general.[29] Consider an abstraction operation Σ that takes physical objects as inputs and generates their *shapes*. This operation would be governed by the identity-criterion saying that if $\Sigma(x,y)$ and $\Sigma(z,u)$ then $y=u$ iff x and z are *similar*. Suppose that $\Sigma(x,y)$. would then require that $\Box_{x,y}\Sigma(x,y)$. But this is not correct: it is not essential to x that x has the shape it has.

To see that fails, suppose (for simplicity) that x is an object in a Euclidean plane. Following Tarski's axiomatization, we take the geometry of the plane to be described using just the notions of *point*, *betweenness*, and *congruence*.

29 Many thanks to an anonymous referee for the challenging questions that led to the correct formulation of .

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Let x be such that if P is any property that applies to all and only the objects similar to x then P is not definable using just the notions of point, betweenness, and congruence.[30] Any such property P thus has to be defined in terms of particular pluralities of points. Let y be such that $\Sigma(x, y)$. If held we would have $\Box_y \forall z(Pz \leftrightarrow \Sigma(z, y))$, for some such property P. But any such property P depends on some particular points; by Chaining, the shape y, too, would depend on some particular points. But this is implausible, the shape should not depend on any particular points. [31]

Let us return to ground. When (**?**; **?**; **?**; **?**) proposed a ground-theoretic account of abstraction principles they did so to defend (or at least explore) an Aristotelian conception of abstracta where the values of an abstraction operation exist because the inputs to these abstraction operations exist. In particular, they were interested in understanding Hume's Principle in this way: a number exists because a plurality with the relevant cardinality exists.

It is important to see that rejecting ground-theoretic criteria of identity does not threaten this Aristotelian conception of abstracta. Consider the case of numbers. Instead of holding that identity criteria specify the *grounds* for 2's being identical with 2 one may simply say that what grounds the *existence* of the number 2 is the existence of some plurality with exactly two members.[32]

Of course, if one adopts a view like this one has to distinguish between the proposition that a exists and the proposition that $a = a$. While this is not uncontroversial, it is the orthodox position in the logic of ground (**?**); moreover, the standard arguments for distinguishing between the proposition that a exists and the proposition that $a = a$ do not turn on how to understand criteria of identity.

If one accepts that the existence of a number is grounded in the existence of some plurality from which that number can be abstracted—and that the existence of a set is grounded in the existence of a plurality from which it is abstracted, and that the existence of a direction is grounded in the existence

30 By cardinality considerations we can prove that there are such objects. There are only countably many shapes that are definable using just *point*, *betweenness*, and *congruence*. But there are uncountably many shapes.

31 It would be interesting to analyze cases like this using Fine's idea of the "essential manifold" (**?**); however, discussion of this has to await another occasion.

32 Crucially not "all pluralities from which the number can be abstracted". The plurality containing just Biden and 2 has exactly two members, but if the existence of 2 is grounded in the existence of that plurality we have cycle of ground. This problem of "auto-abstraction" raises similar problems to the puzzles of ground (**?**). For an account of auto-abstraction in particular, see (**?**); for general approaches to the puzzles, see (**?**; **?**; **?**; **?**).

of a line from which it is abstracted etc—one should hold that this is no accident, but is rather explained by the nature of numbers (sets, direction). As representative example one should therefore accept the following essentialist claim about ground:

$$\Box_{\mathrm{SetOf}} \forall xx \forall y (\mathrm{SetOf}(xx, y) \rightarrow (Exx < Ey))$$

(Here we write $<$ for strict full ground.)

The defenders of a ground-theoretic understanding of Hume's Principle were not just interested in providing grounds for identities. As is well known, Hume's Principle suffices—in the presence of second-order logic and assuming the standard Fregean definitions of the successor relation, zero, and the property of being a natural number—to derive the second-order Dedekind-Peano axioms. The ground-theoretic understanding of Hume's Principle thus—together with standard assumptions in the logic of ground—allows us to characterize the grounds for all arithmetical truths.

The ground-theoretic understanding of Hume's Principle is, however, not required for this. Uniform views about the grounds for identities—like those of Litland and Rubenstein—also allow us to provide grounds for all the arithmetical truths. To see this, observe that if one bases arithmetic on Hume's principle all arithmetical notions are ultimately defined in terms of the cardinality-abstraction operation #, identity and distinctness, and the other logical notions. The uniform views about the grounds for identity and distinctness facts provide grounds for *all* such facts, not just the arithmetical ones. The grounds for the other arithmetical claims are then determined in accordance with the definitions of the properties involved in them.[33]

To illustrate, consider the definition of the successor relation. Let us write $\#(xx, y)$ to mean that y is the result of cardinality-abstracting on xx; and if xx is a plurality and y is an object let us write $[xx, y]$ for the plurality that contains exactly the xx and in addition y. The successor relation S is defined in terms of # as follows: for all a, b we have Sab iff $\exists xx \exists y (y \not\prec xx \wedge \#(xx, a) \wedge \#([xx, y], b))$. The (immediate) grounds for Sab will thus be triples of facts of the form: $y \not\prec xx, \#(xx, a), \#([xx, y], b)$.

Finally, we should show how the essentialist account of criteria of identity interacts with the uniform views about the grounds of identity facts mentioned

33 How this is done of course depends on which logical grounding principles are accepted. But we can assume that these logical grounding principles are common ground—no pun intended.

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above. We illustrate this using Litland's account, but the relationship is similar on other uniform accounts.

Consider two distinct objects a, b that are in fact F and consider the propositions $a = a$ and $a \neq b$. We can ask what lies in the natures of these propositions considered just as identity and distinctness propositions. This comes down to what can be derived from them using just the facts about the essence of the identity-relation as auxiliary premisses. On Litland's view the essence of the identity relation itself is exhausted by identity facts being uniquely zero-grounded and distinctness facts being uniquely zero-grounded. Thus the only thing that lies in the nature of $a = a$ considered just as an identity fact is that it is uniquely zero-grounded; and the only that lies in the nature of $a \neq b$ considered just as a distinctness fact is that it is uniquely zero-grounded. But there is much more to the natures of these propositions than their being uniquely zero-grounded: by the Chaining Principle anything that is essential to a will be essential to $a = a$ (and similarly for a, b and $a \neq b$). The identity criterion for Fs help pin down the richer nature of the Fs. Thus it will be essential to the proposition that $a = a$ that $R_F aa$ and it will be essential to the proposition that $a \neq b$ that $\neg R_F ab$.

4 Conclusion

We have argued against a ground-theoretic construal of criteria of identity and in favor of an essentialist construal; we have argued that functional formulations of two-level criteria of identity should be eschewed; and we have argued that the essentialist account captures everything worth salvaging in the ground theoretic account. Let us end by relating what we have done to one of Frege's famous discussions of criteria of identity.

In his famous discussion of Hume's Principle Frege writes:

> We are proposing not to define identity specifically for this case , but to use the concept of identity, taken as already known, as a means for arriving at that which is to be regarded as being identical (**?**).

The position we have developed here is a metaphysical analogue of Frege's: the identity relation is *used* in saying something about the natures of the values of certain abstraction operations, but the identity criteria do not tell us something about the nature of the identity relation itself.[34][*]

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34 Note that if the identity relation is used in specifying the nature of what it is to be an F in general, then the property of being an F depends on the identity relation. It may be worth noting that it is possible to specify the nature of being an F and even the nature of abstraction operations without using the identity relation at all. In a language governed by the Wittgensteinean Variable Convention distinct bound variables are constrained to take distinct values. (See (**?**) for details.) In such a language one can state criteria of identity without using the identity-relation. A one-level criterion like can be stated as follows:

$$\Box_F \forall x \forall y (Fx \land Fy \to \neg R_F(x,y))$$

A two-level identity criterion like can be stated as follows:

$$\forall x \forall y \forall z \forall u (F(x,z) \land F(y,u) \to \neg R_F(x,y))$$

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